Excellence Cluster Universe

N-body algorithms in GANDALF

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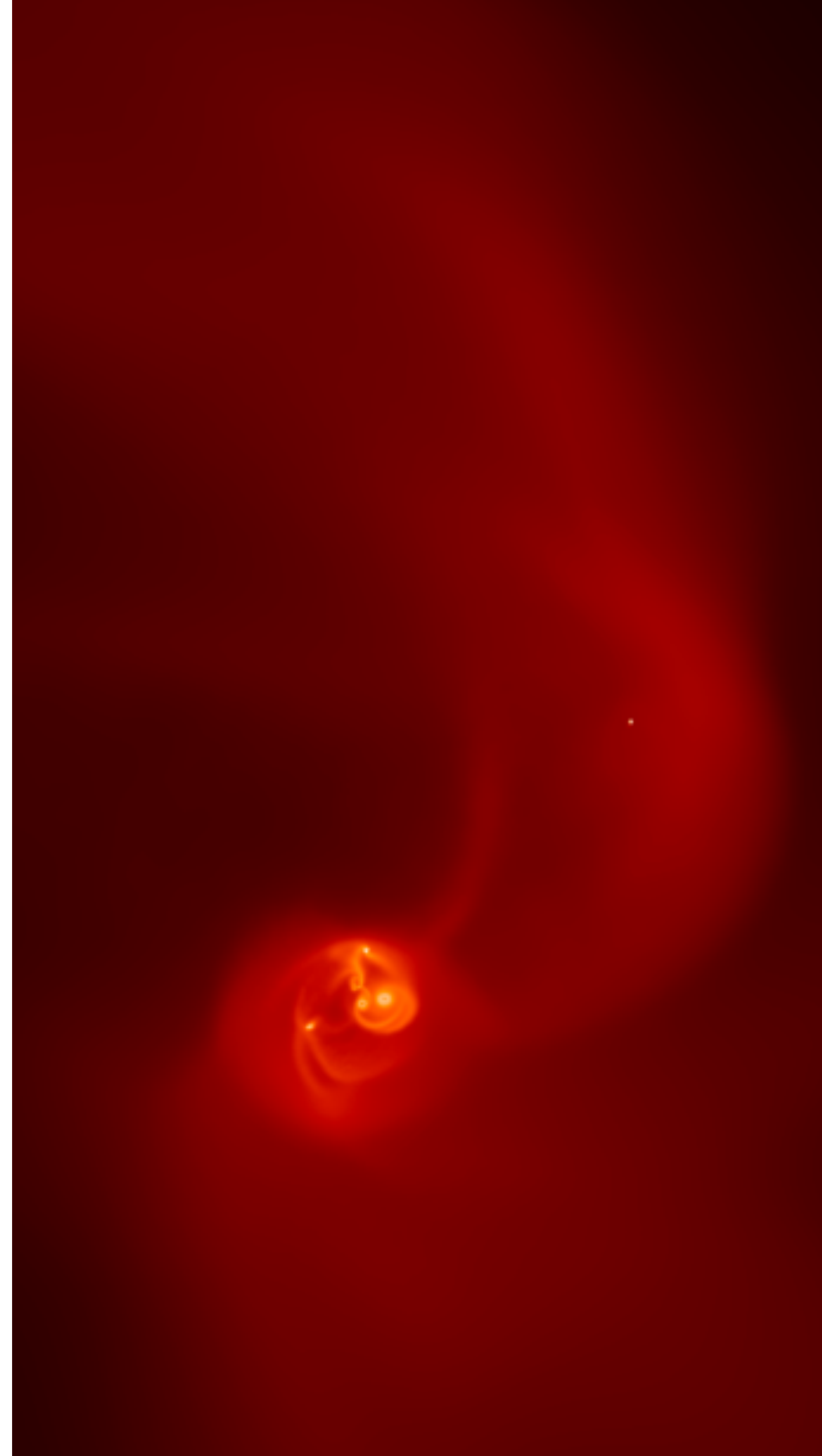
30th October 2015



Collisional vs. Collisionless N-body dynamics

- N-body algorithms are usually divided up into two main classes :
  - **Collisional** : N-body particles are central point masses which can have strong 2-body interactions (e.g. stellar encounters)
    - NBODY6, Starlab/kira
  - **Collisionless** : N-body particles have a smoothed potential so only feel long-range potential forces (e.g. cold-dark matter fluid)
    - GADGET 2/3, GASOLINE
- Both 'versions' of N-body simulations can be realised in GANDALF
- However, the collisional N-body dynamics is **only realised designed for relatively small N-body systems** and not for large-N systems (e.g. the million body problem)

Simple collisionless N-body integrators

- **Collisionless** N-body integrators in GANDALF use the same algorithms as the SPH particles, i.e.
  - Leapfrog kick-drift-kick (i.e. lfkdk)

$$\mathbf{r}_i(t + \Delta t) = \mathbf{r}_i(t) + \mathbf{v}_i(t)\,\Delta t + \frac{1}{2}\,\mathbf{a}_i(t)\,\Delta t^2$$

$$\mathbf{v}_i(t + \Delta t) = \mathbf{v}_i(t) + \frac{1}{2}\left(\mathbf{a}_i(t) + \mathbf{a}_i(t + \Delta t)\right)\,\Delta t$$

  - Leapfrog drift-kick-drift (i.e. lfdkd)

- These integrators are **symplectic**, i.e. have very good conservation properties, particularly angular momentum

Simple collisionless N-body integrators

- **Simplest way** to simulate collisionless N-body is to use SPH particles with self gravity but hydro_forces switched off!

```
hydro_forces = 0
self_gravity = 1
```

- Developing multi-species in GANDALF in order to have cdm particles, i.e. self-gravtiating but no hydro forces, as well as hydro particles

```
template <int ndim>
struct Particle
{
  bool active;                        ///< Flag if active (i.e. recompute step)
  bool potmin;                        ///< Is particle at a potential minima?
  int iorig;                          ///< Original particle i.d.
  int itype;                          ///< SPH particle type
  etc..
};
```

```
part.itype = gas;
part.itype = cdm;
```

Simple collisional N-body integrators

- **Collisional** N-body integrators are more demanding because
  - Stars may have rather violent 2-body (or 3-body) interactions
  - Requires much higher accuracy with the integrations

- Simplest integrators are the same as the collisionless code
  - Leapfrog kick-drift-kick (i.e. lfkdk)
  - Leapfrog drift-kick-drift (i.e. lfdkd)

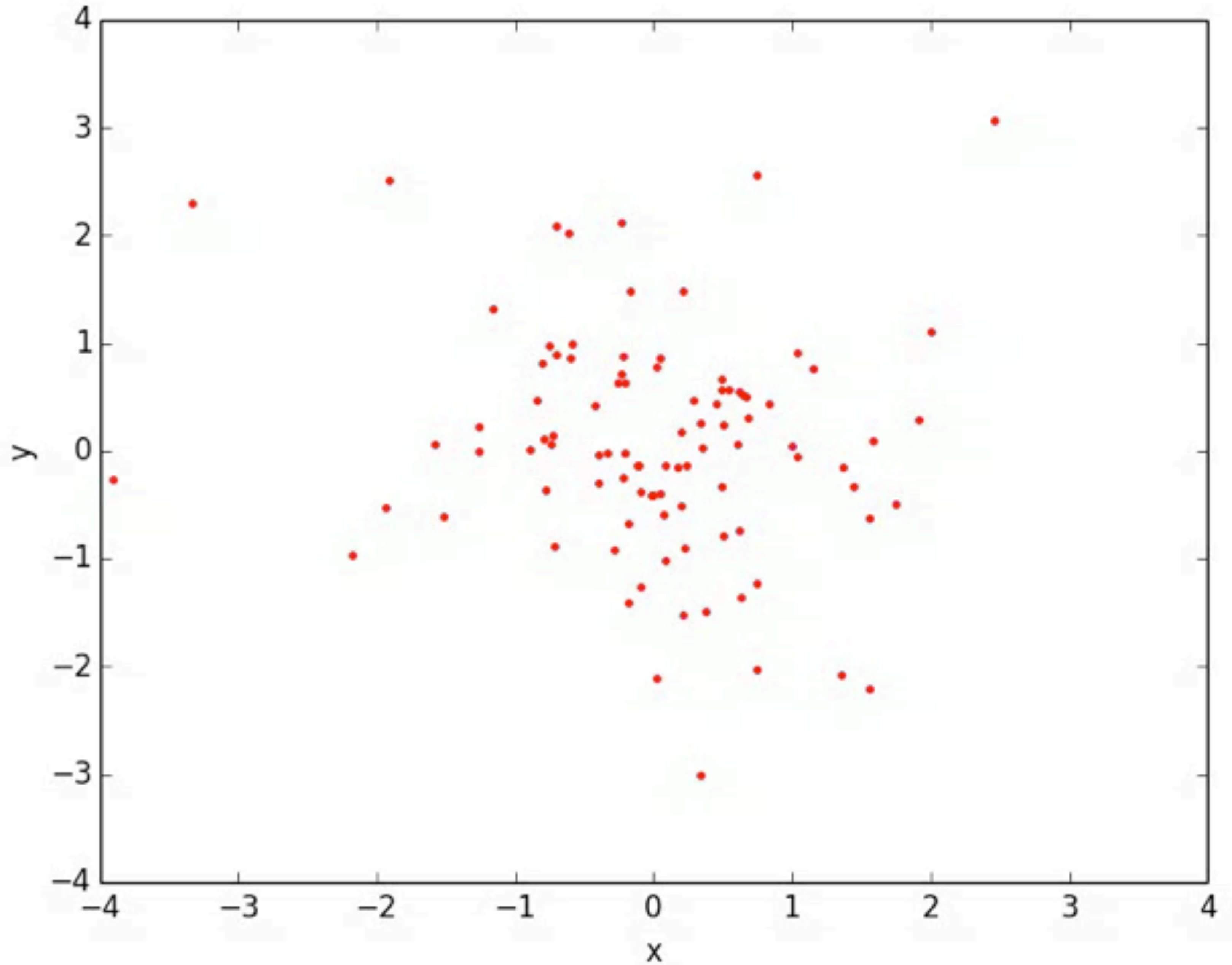
More sophisticated N-body integrators

- For more accuracy, we can use :
  - 4th, 6th and 8th-order Hermite scheme (Makino & Aarseth 1992)
  - KS-regularisation
- Hermite schemes compute both the force AND the force derivative

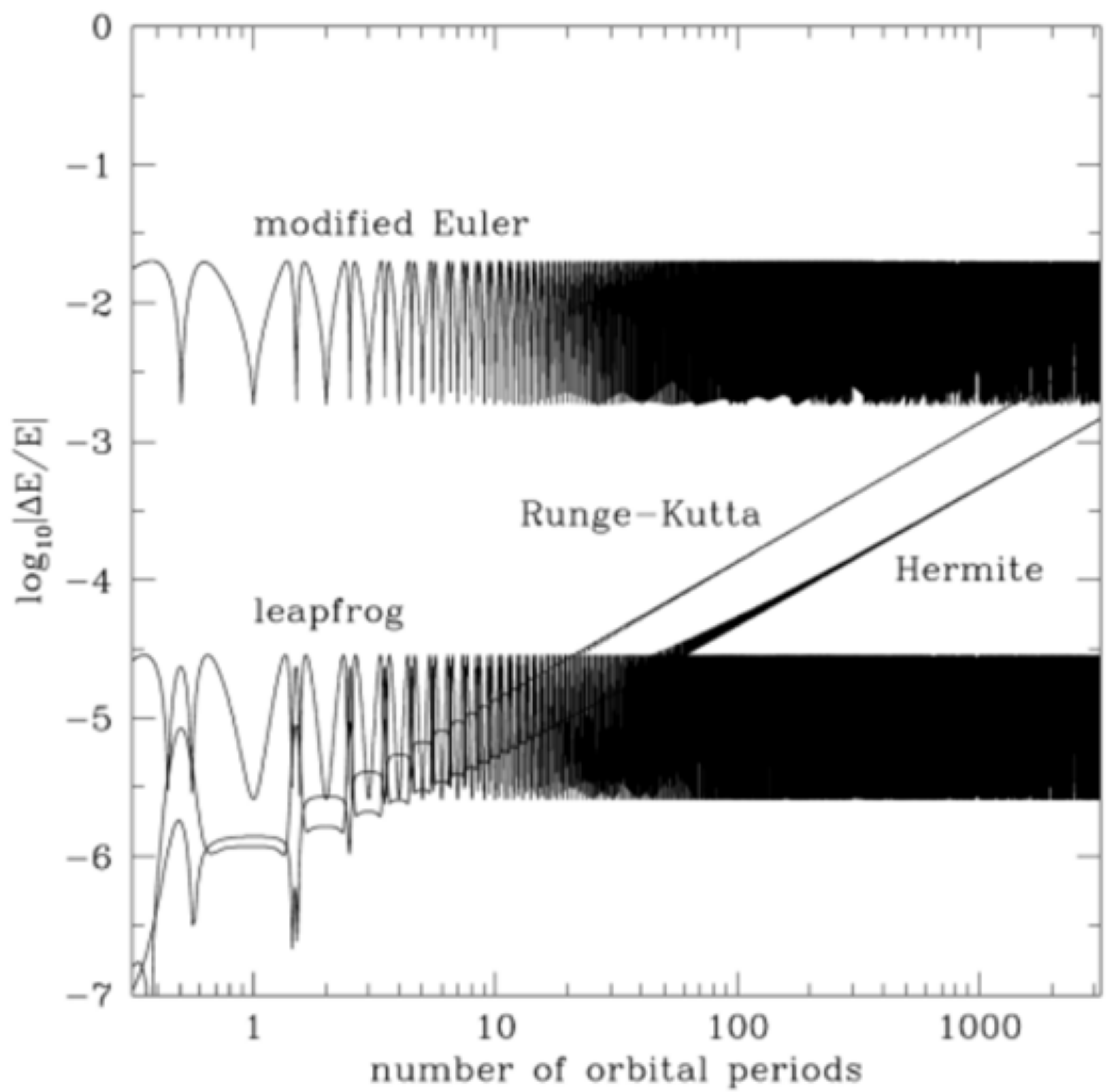
$$\mathbf{a}_s = -G \sum_{t=1}^{N} m_t\, \phi'(\mathbf{r}_{st}, \overline{h}_{st})\, \hat{\mathbf{r}}_{st} - G \sum_{i=1}^{N} m_i\, \phi'(\mathbf{r}_{si}, \overline{h}_{si})\, \hat{\mathbf{r}}_{si}$$

$$\dot{\mathbf{a}}_s = -G \sum_{t=1}^{N} \frac{m_t\, \phi'(\mathbf{r}_{st}, \overline{h}_{st})}{|\mathbf{r}_{st}|} \mathbf{v}_{st} + 3\, G \sum_{t=1}^{N} \frac{m_t\, (\mathbf{r}_{st} \cdot \mathbf{v}_{st})\, \phi'(\mathbf{r}_{st}, \overline{h}_{st})}{|\mathbf{r}_{st}|^3} \mathbf{r}_{st}$$
$$- 4\, \pi\, G \sum_{t=1}^{N} \frac{m_t\ (\mathbf{r}_{st} \cdot \mathbf{v}_{st})\, W(\mathbf{r}_{st}, \overline{h}_{st})}{|\mathbf{r}_{st}|^2} \mathbf{r}_{st}\, .$$

A simple example : A plummer sphere (N = 100)



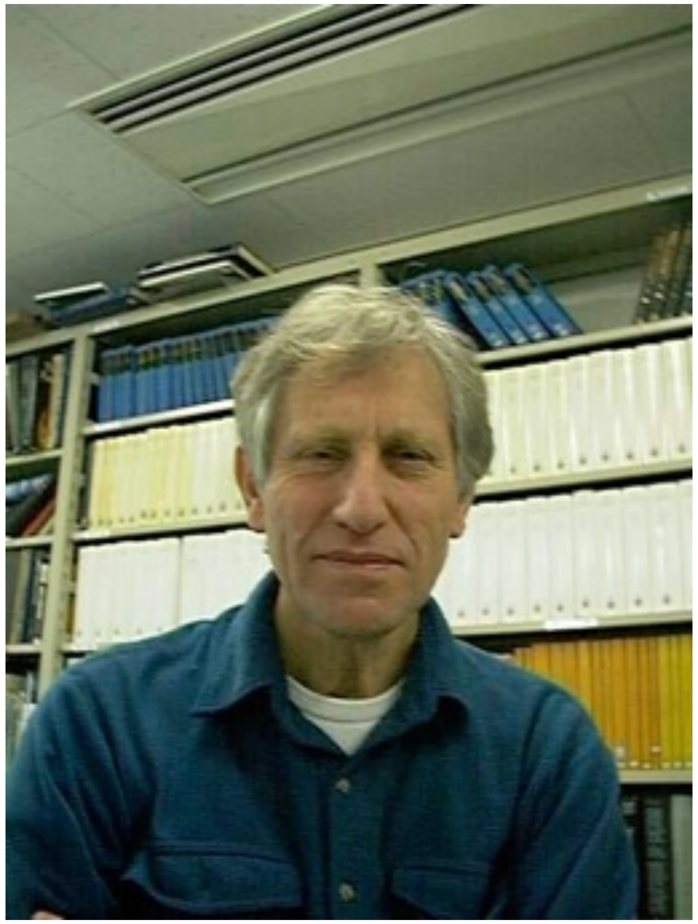
Energy errors in N-body codes



Galactic Dynamics (Binney & Tremaine 2008)

What about ‘Regularisation’?

- **KS-Regularisation** is a powerful technique used in some N-body codes to :
  - (i) allow very accurate integration of very close 2-body encounters
  - (ii) therefore eliminate the need for softening/smoothing of grav. forces
- Some reasons not to use it
  - **Extremely** complicated
  - Hard to combine other physics (e.g. gas forces)
  - There are alternatives these days, not quite as accurate but much easier to implement
- Will I get hunted down by Sverre Aarseth if I don’t use it??
  - Hopefully not



Sub-systems

- If binary or higher-order multiple sysyems form, then the simulation may progress slower and slower
  - **Spends a lot of CPU effort integrating the binary system** with short timesteps as the **rest of the simulation proceeds very slowly**
  - Most of the time, **the binary motion can be isolated and simulated as a separate system** (with or without external perturbations)
- If a binary is identified (as in the previous slide), then
  - Binary motion is integrated separately
  - Rest of simulation interacts with centre-of-mass of binary

Hydrodynamics + N-body

- GANDALF employs a hybrid scheme for modelling the evolution of a gaseous stellar cluster
  - Gas is modelled with SPH particles using 2nd order Leapfrog scheme
  - N-body particles are modelled with 4th-order Hermite scheme
  - Derived **coupling terms that maintains energy conservation**

Possible challenges to hybrid scheme

- **N-body codes usually require high accuracy** (e.g. total energy conserved to less than 0.001% accuracy), but hydro-codes usually operate with much higher error tolerances.

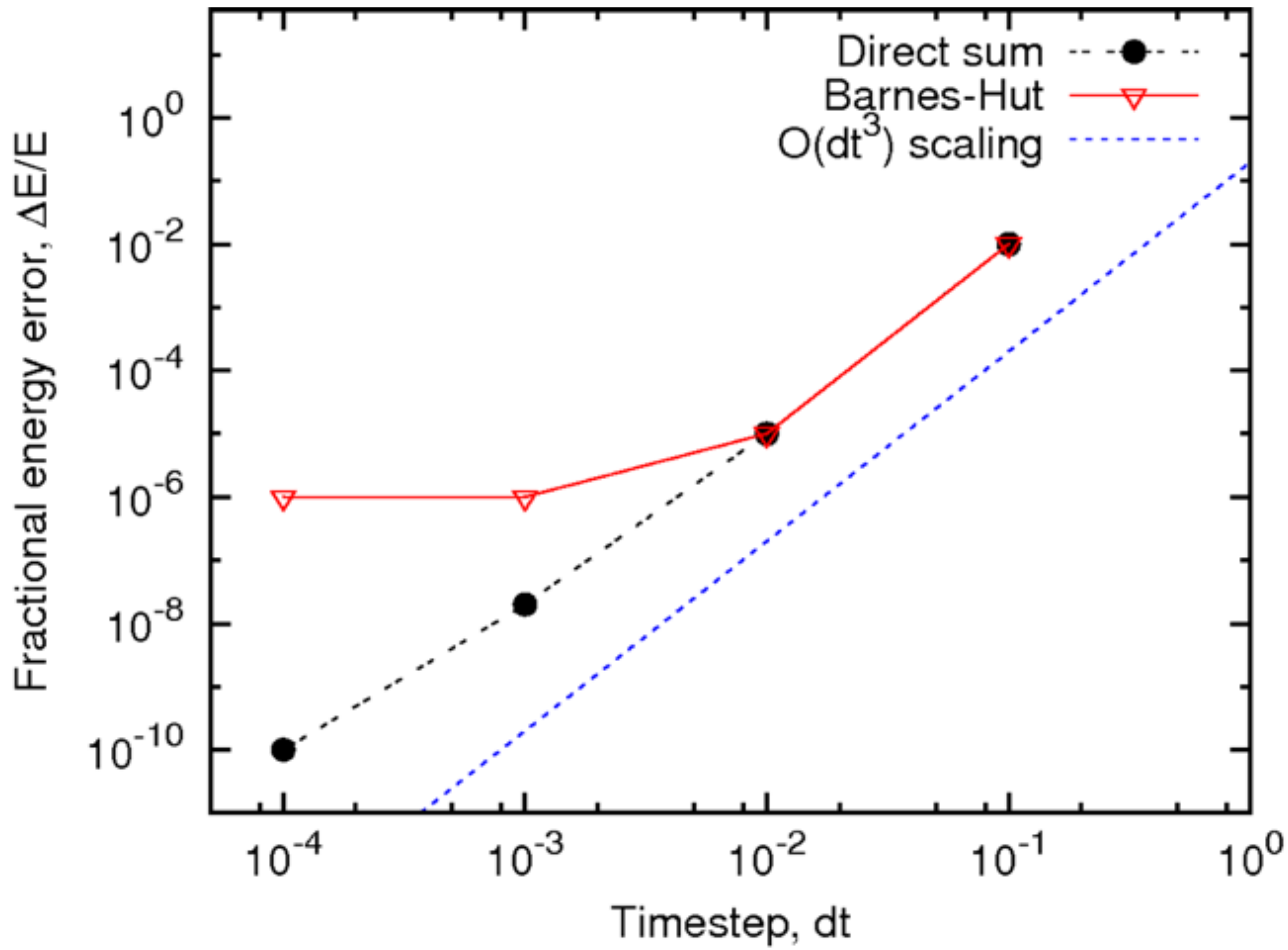
> challenges. A simple workaround has been proposed by [200]. For each and every simulation the conservation of energy, momentum and angular momentum should be monitored. Reducing the time step size and increasing the force accuracy, say, if a tree is used for gravity, should improve the conservation properties. A correct code should ensure conservation to better than 1% over several thousand time steps.

Rosswog (2009)

- However, modern SPH schemes derived via Lagrangian mechanics can, in principle, conserve momentum, angular momentum and energy to rounding error given a robust integration scheme.

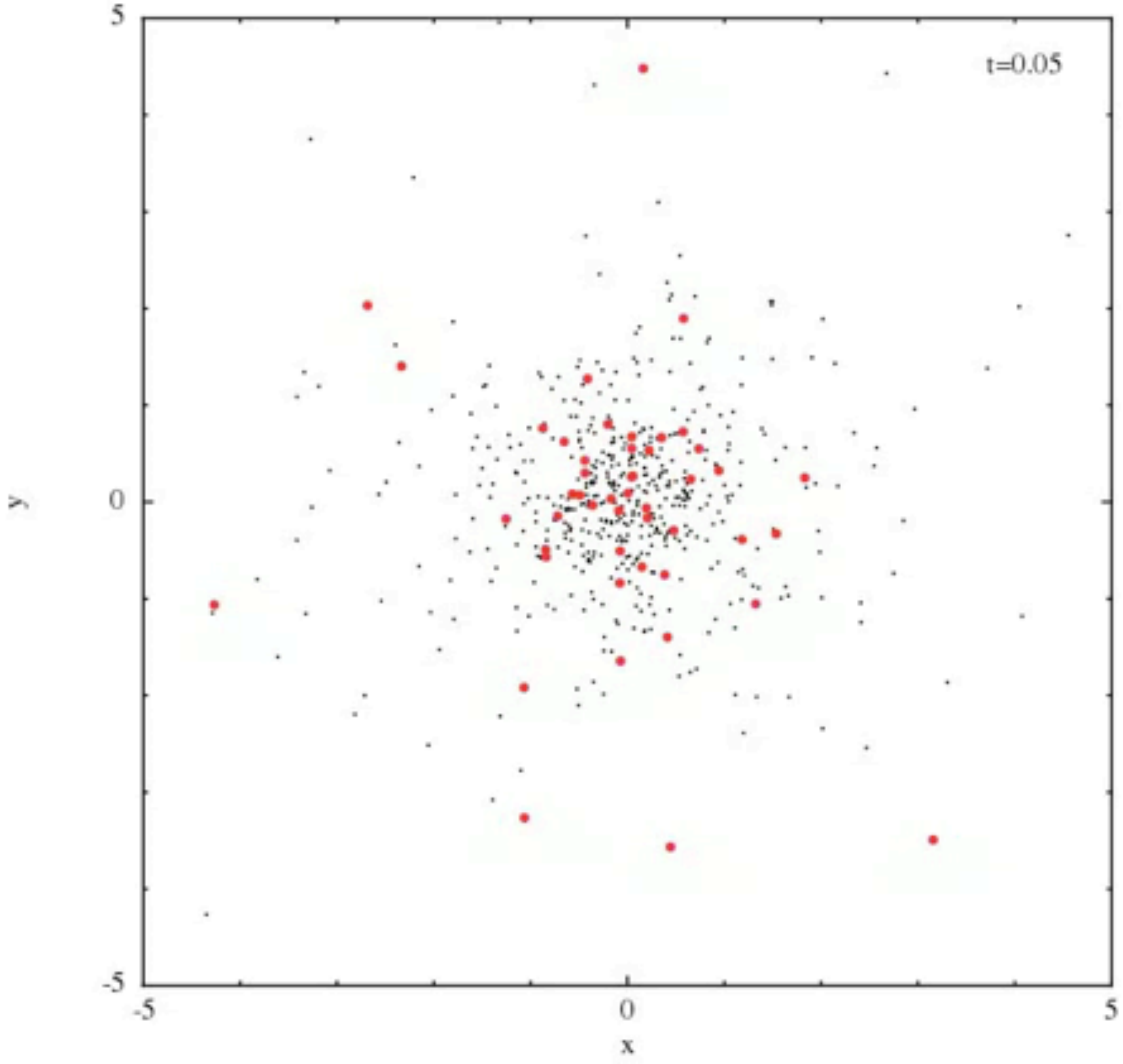
Errors in SPH/N-body codes

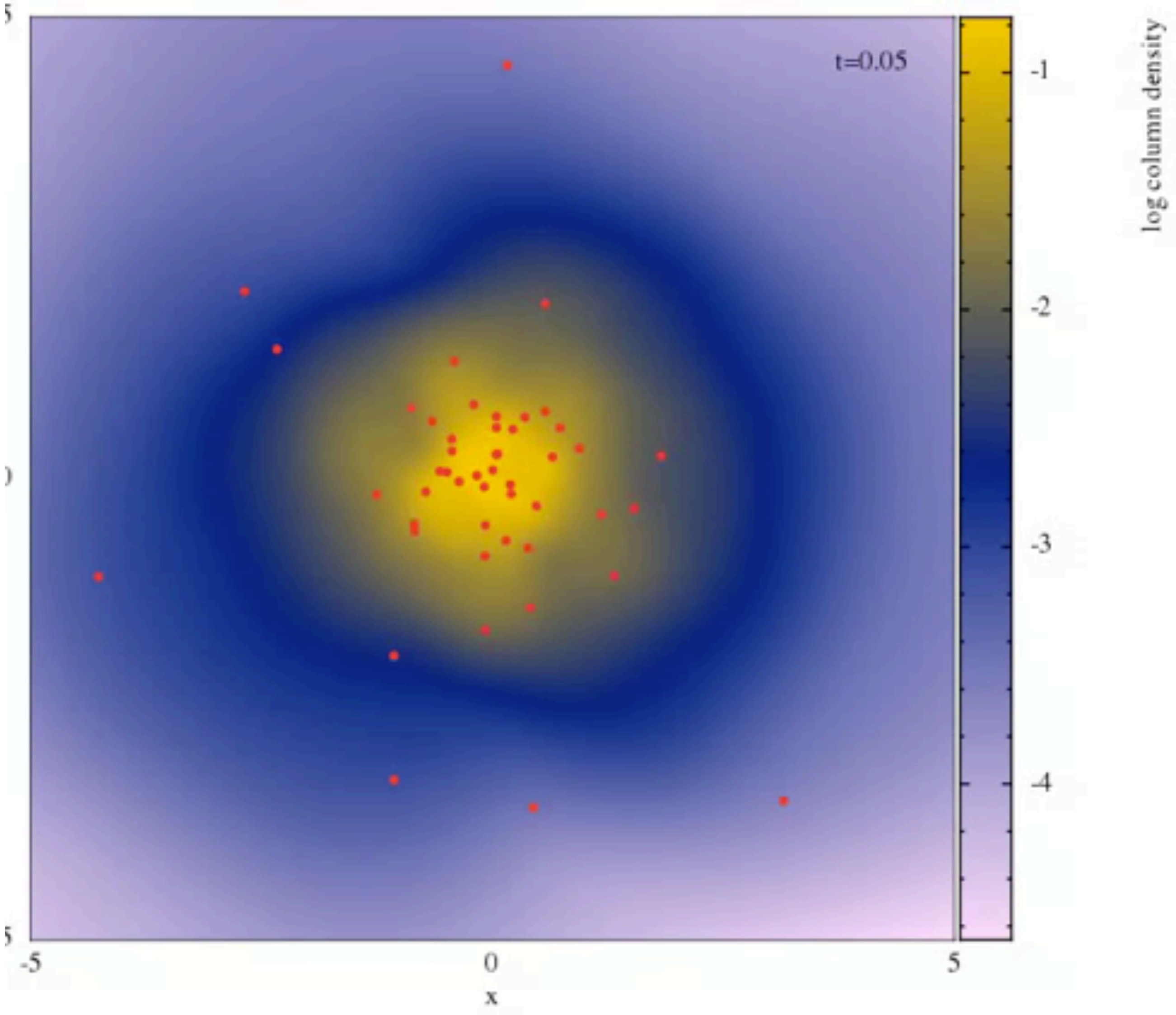
- Integration (truncation) error
  - SPH - 2nd-order Leapfrog
  - N-body - 4th-order Hermite
- Block timesteps
- Gravity tree errors



Gaseous Plummer spheres

- A Plummer sphere can be combined with a n=5 polytrope to produce a stable ‘gaseous cluster’.





Modelling star formation : Sink particles

- Modelling how low-density gas collapses into stars is a very expensive process
  - Can perhaps investigate a single star in detail
  - **Almost impossible with current capabilities to model a cluster of fully formed stars**

- Bate, Bonnel & Price (1995) introduced **dynamical sink particles**, to mimic the formation of a star and to capture the effects of any subsequent accretion
  - Sinks are created like little black holes / vacuum cleaners that sweep up any gas that enters it
  - Allows simulations to run fast enough to follow large-scale cluster formation

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Sink particles : formation criteria

- The choice of formation criteria is crucial for obtaining converged simulations
- We use the following criteria
  - Exceeds a density threshold

    $$\rho_i > \rho_{\rm SINK}$$

  - Gravitational potential minima

    $$\phi_i \quad < \quad {\rm MIN}\,\{\phi_j\}$$

  - Doesn't overlap with existing sink

    $$|\mathbf{r}_i - \mathbf{r}_{s'}| \quad > \quad X_{\rm SINK}\, h_i \,+\, R_{s'}$$

- There's an additional criterion which should be implemented soon
  - Hills sphere criteria

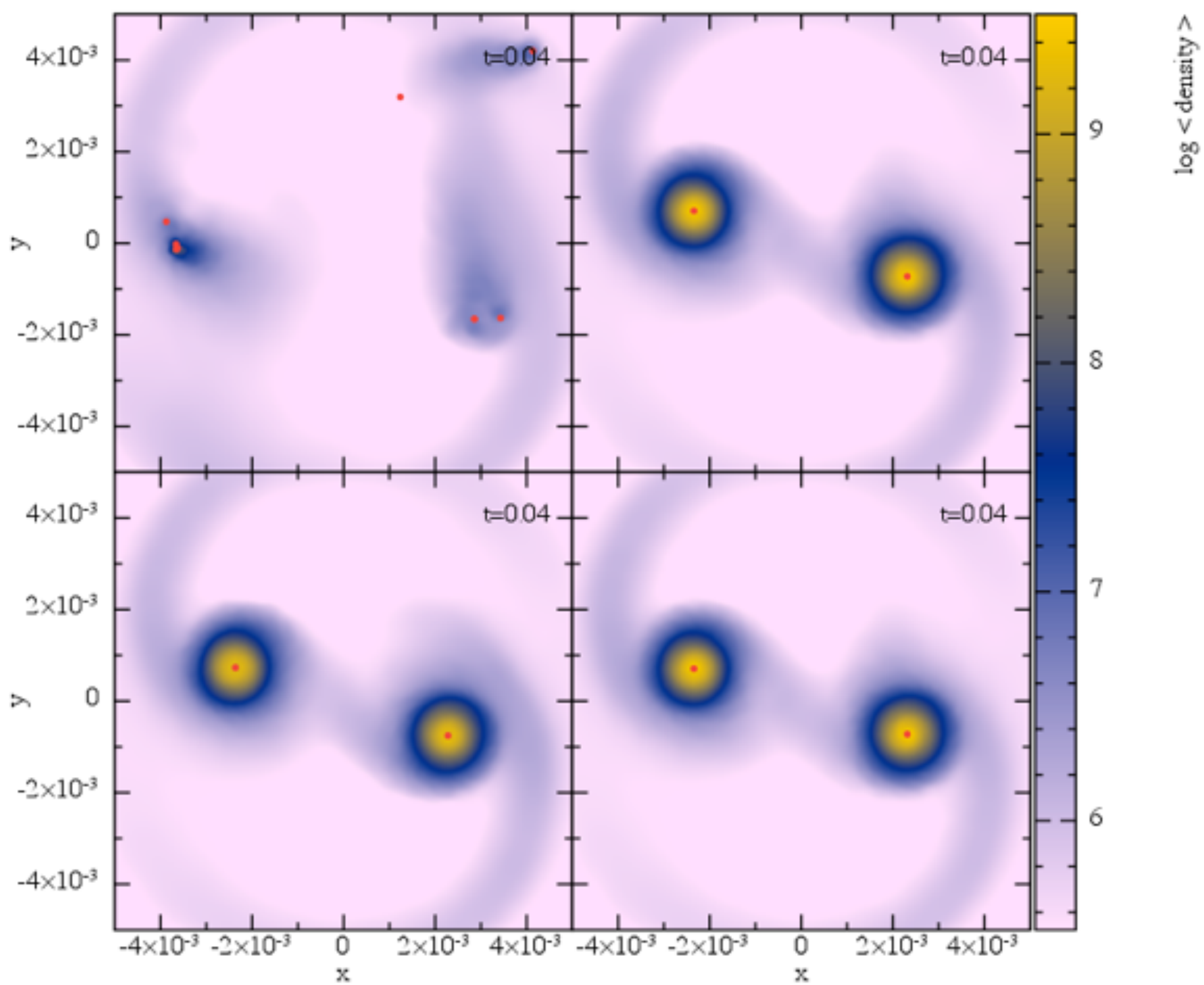
    $$\rho_i \quad > \quad \rho_{\rm HILL} \quad \equiv \quad \frac{3\,X_{\rm HILL}\;(-\Delta\mathbf{r}_{is'}\cdot\Delta\mathbf{a}_{is'})}{4\,\pi\,G\,|\Delta\mathbf{r}_{is'}|^2}$$

Sink particles : formation criteria

Low sink density High sink density

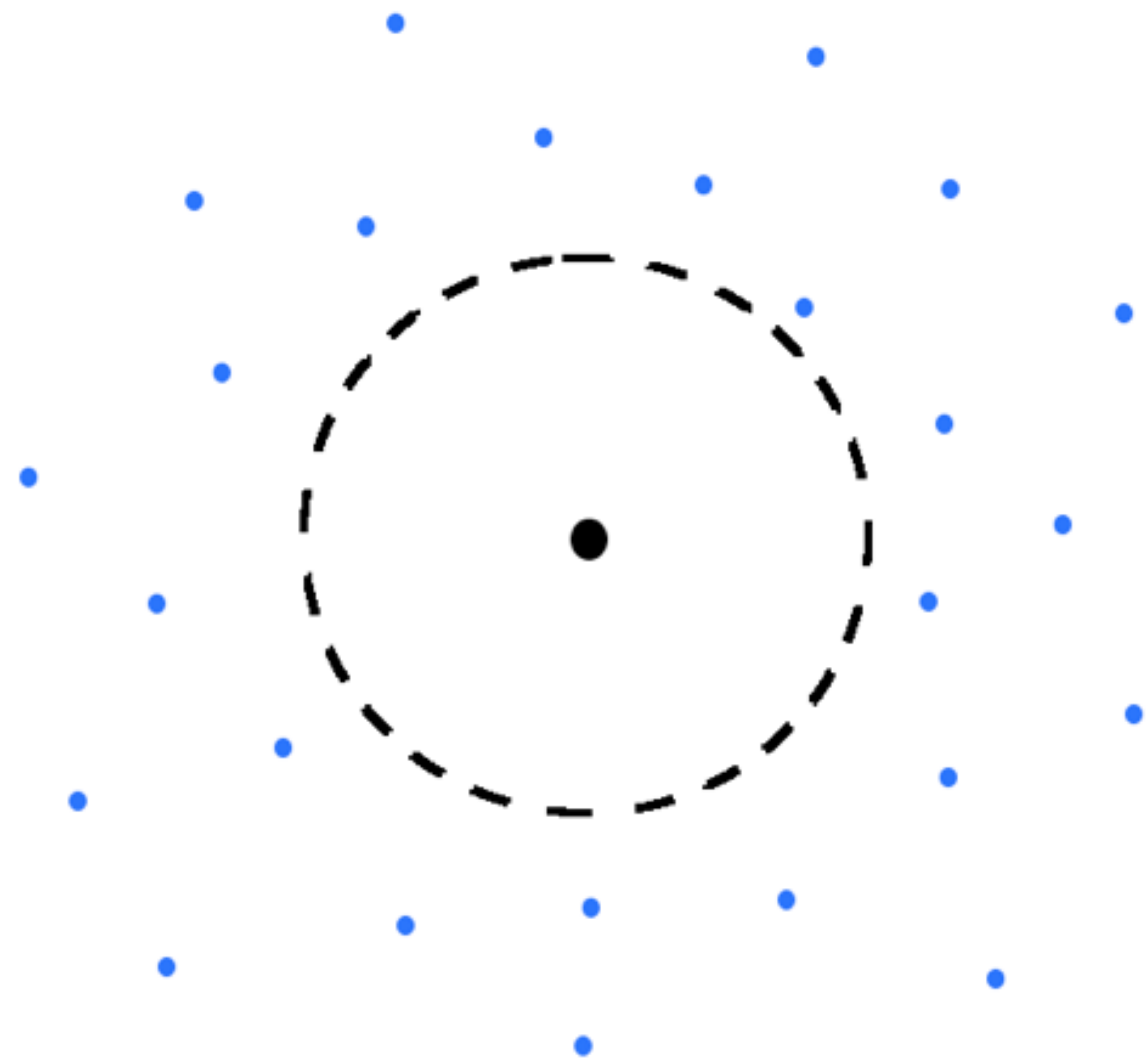
Density criterion

Density & potential minimum criteria



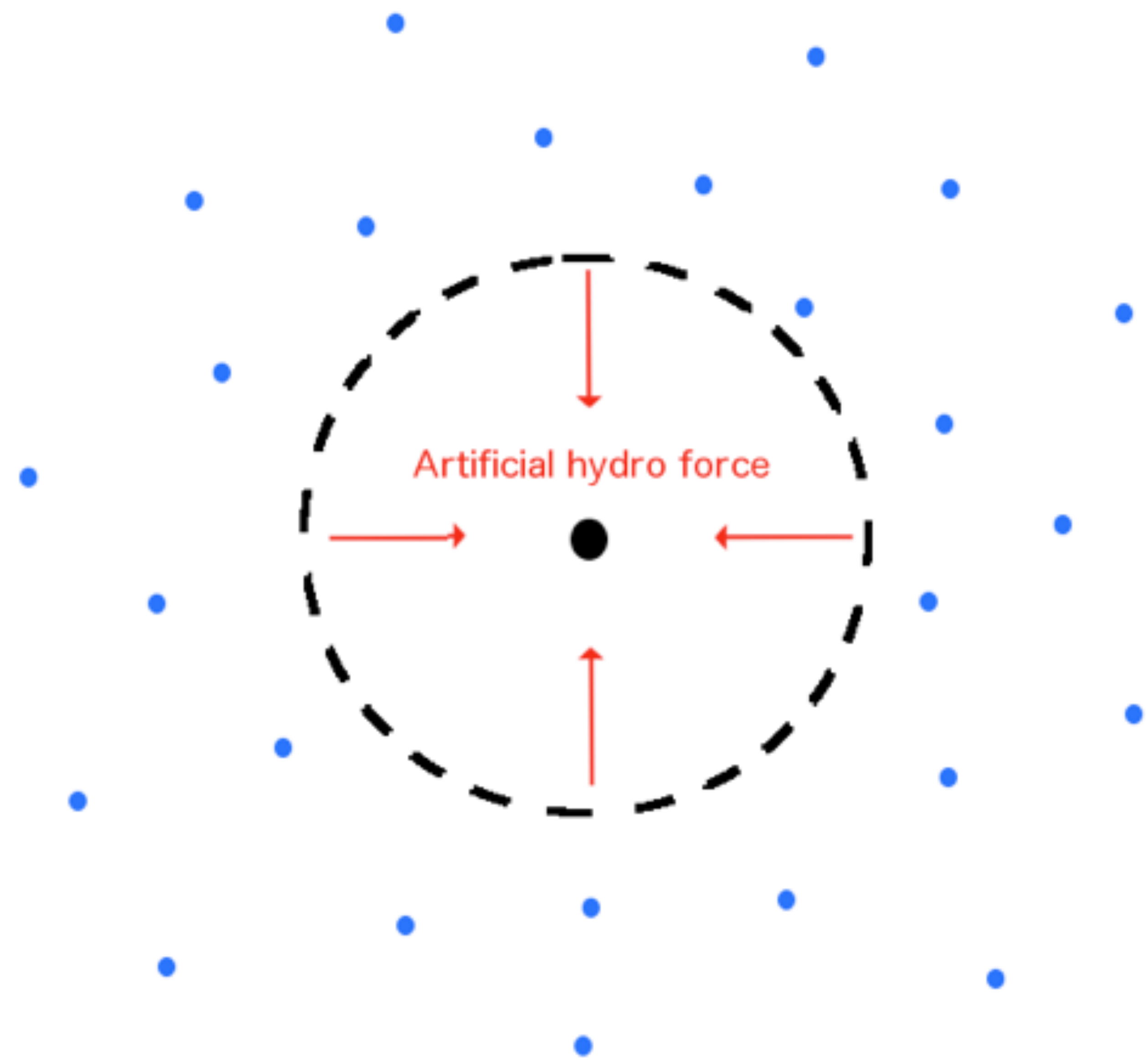
Modelling accretion

- Accretion is modelled by removing particles from the simulation that
  - Enter the sink accretion radius
  - Are gravitationally bound to the sink
- Generally leads to an empty ‘exclusion’ zone inside the sink that is devoid of any SPH particles



Artificial boundary forces

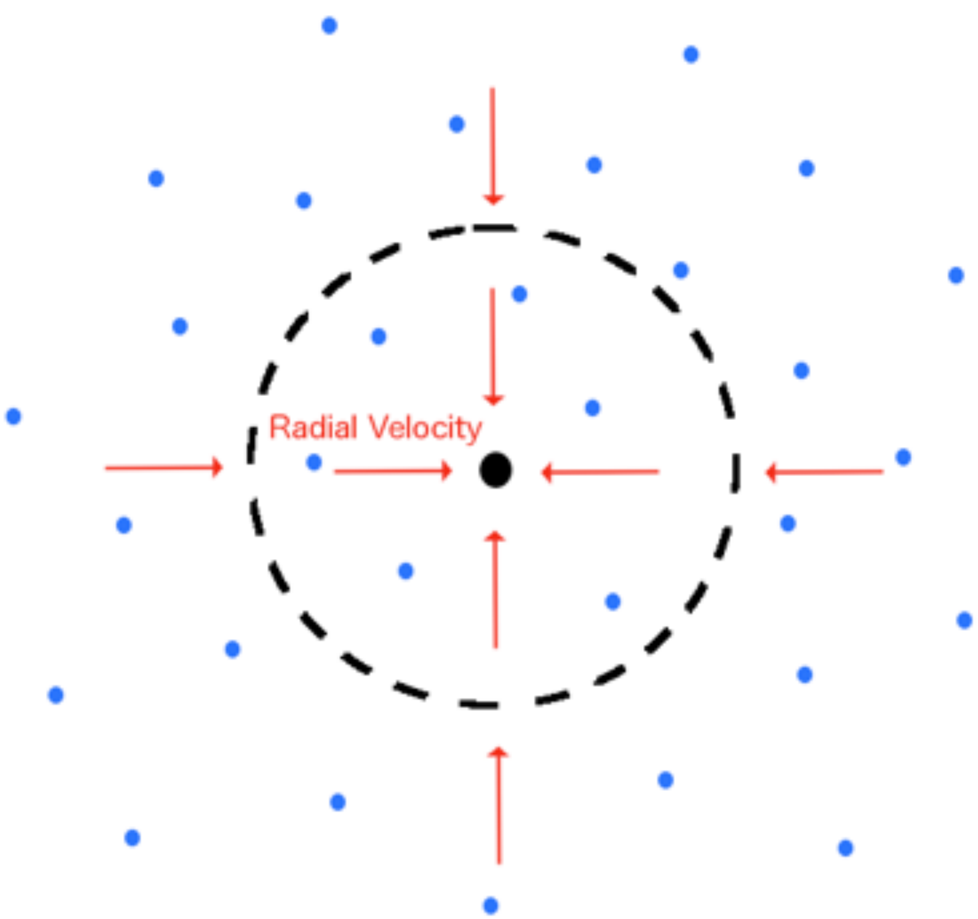
- Particles just outside the accretion radius see no neighbours inside the sink
  - Discontinuous sampling of density field
  - All SPH properties are incorrect, in particular the hydro forces
  - Leads to artificial outward pressure gradient, and therefore artificial inward hydro force
- BBP95 originally suggested using some correction terms to account for missing neighbours
  - Does not work so well and is not used any more (as far as I know)



Spherical and disc accretion

- For sub-grid accretion model, we consider two limiting cases

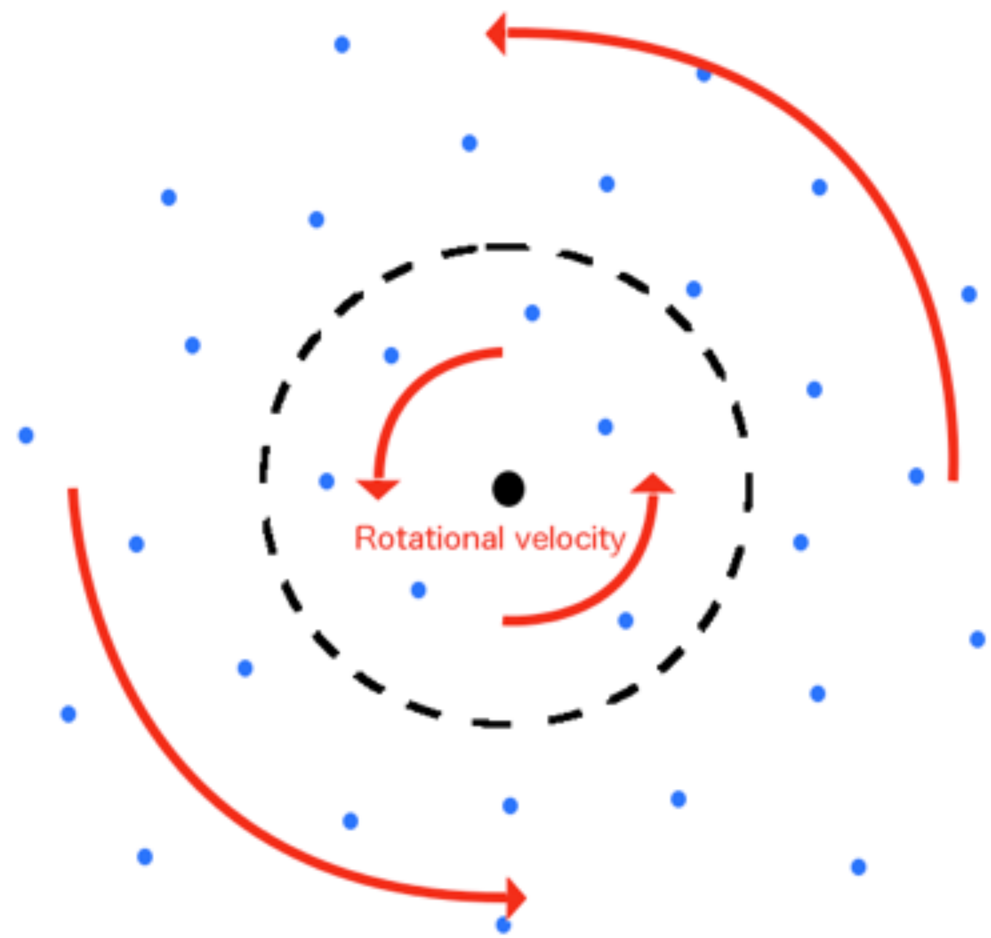
Spherical accretion



$$\dot{M}(r) = -4\pi r^2 \rho(r) v_{\rm RAD}(r)$$

$$\langle t_{\rm RAD} \rangle_s = \frac{\sum_j \{m_j\}\, \mathcal{W}}{4\pi \sum_j \{|\Delta \mathbf{r}_{js}| \Delta \mathbf{r}_{js} \cdot \Delta \mathbf{v}_{js} m_j W(|\Delta \mathbf{r}_{js}|, H_s)\}}$$

Disc accretion



$$t_{\rm SS} = \frac{(G\, M_*\, R_d)^{1/2}}{\alpha_{\rm SS}\; a^2}$$

$$\langle t_{\rm DISC} \rangle = \frac{(GM_s)^{1/2}}{\alpha_{\rm SS}\, \mathcal{W}} \sum_j \left\{ \frac{|\Delta \mathbf{r}_{js}|^{1/2} m_j W(|\Delta \mathbf{r}_{js}|, H_s)}{\rho_j a_j^2} \right\}$$

Computing the accretion timescale

- We compute the ratio of rotational energy to gravitational energy of particles inside the sink as an indicator of which limiting case is applicable

$$f = \text{MIN}\left\{\frac{2E_{\text{ROT}}}{|E_{\text{GRAV}}|}, 1\right\}$$

- If f = 1, particles are in **rotational equilibrium** : $t_{\text{ACC}} \to \langle t_{\text{DISC}} \rangle_s$
- If f = 0, particle motion is **purely radial** : $t_{\text{ACC}} \to \langle t_{\text{RAD}} \rangle_s$
- To deal with intermediate cases that also give the correct limiting behaviour, we use

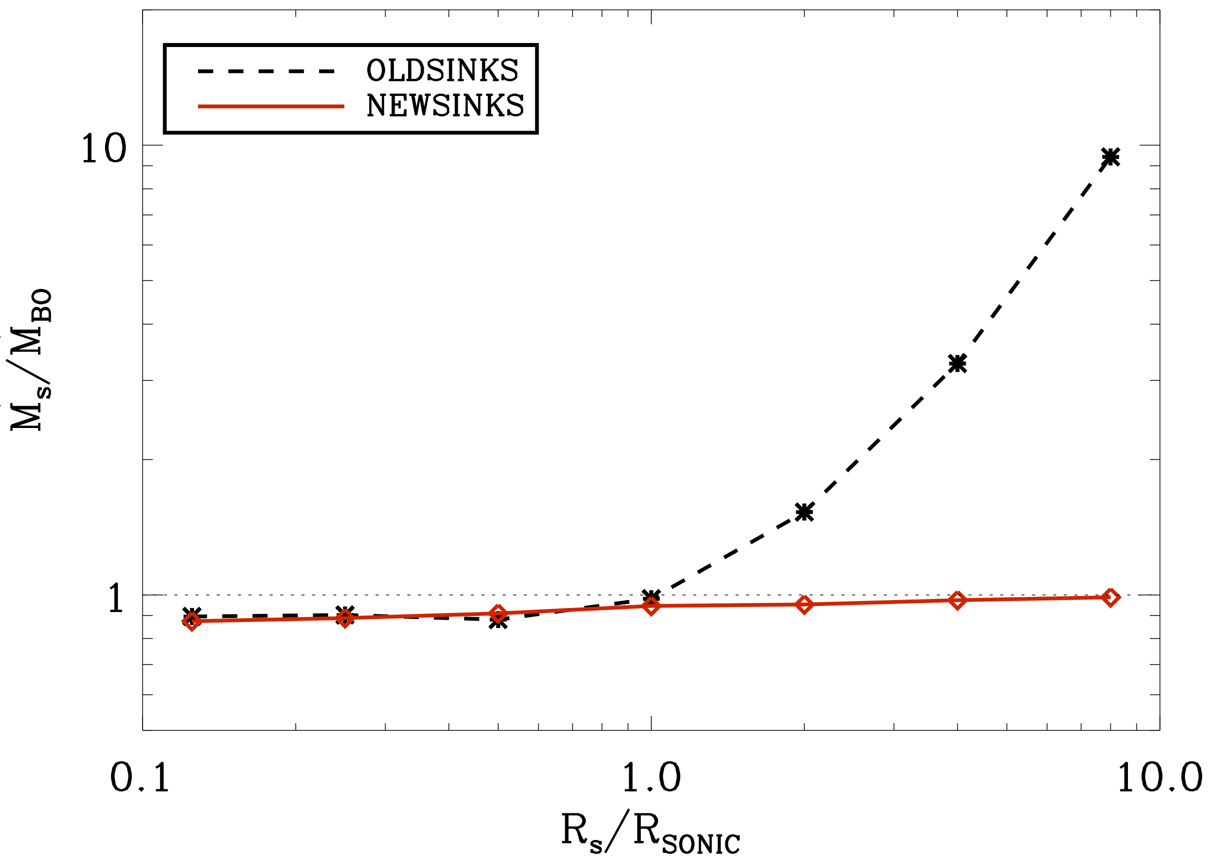
$$t_{\text{ACC}} = \langle t_{\text{RAD}} \rangle_s^{(1-f)} \, \langle t_{\text{DISC}} \rangle_s^{f}$$

- The mass accreted in the current timestep is then

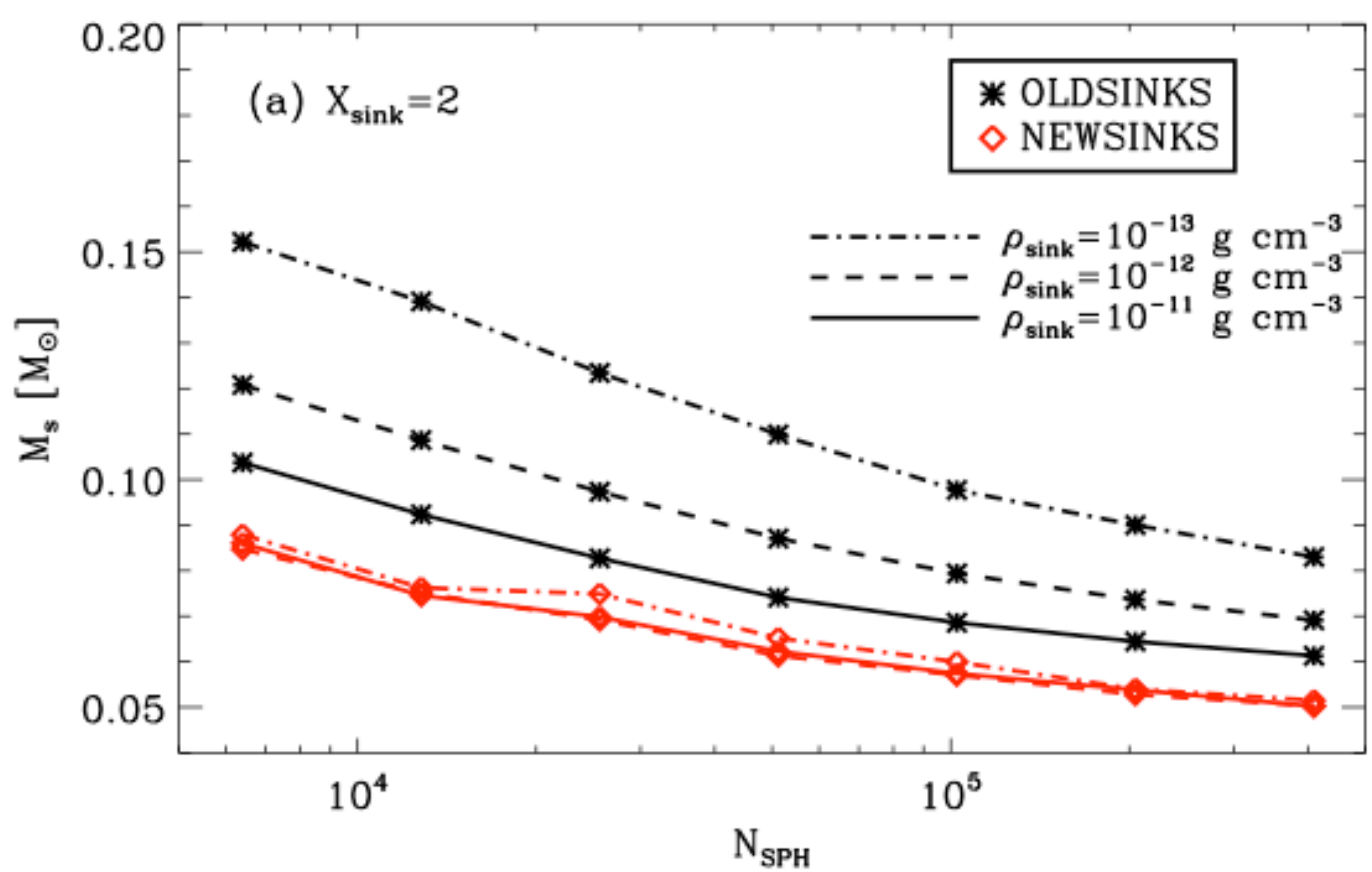
$$\delta M_{\text{ACC}} = M_{\text{INT}} \left[1 - \exp\left(-\frac{\delta t_s}{t_{\text{ACC}}}\right)\right]$$

Bondi accretion (Spherical accretion)

- In Bondi accretion, the sonic point defines the radius where the inflow velocity is equal to the local sound speed
- For **large radii, the inflow is subsonic** (both hydro and gravity forces important)
- For small radii, the inflow is supersonic (only gravity important)
- Old sinks are correct for small radii since the lack of hydro forces is unimportant. For large radii, the lack of hydro forces leads to incorrect accretion rates
- New sinks give correct accretion rates for all sink radii
- Note : For monatomic gases, the sonic radius is zero. Therefore, for old sinks the accretion rate is always wrong.

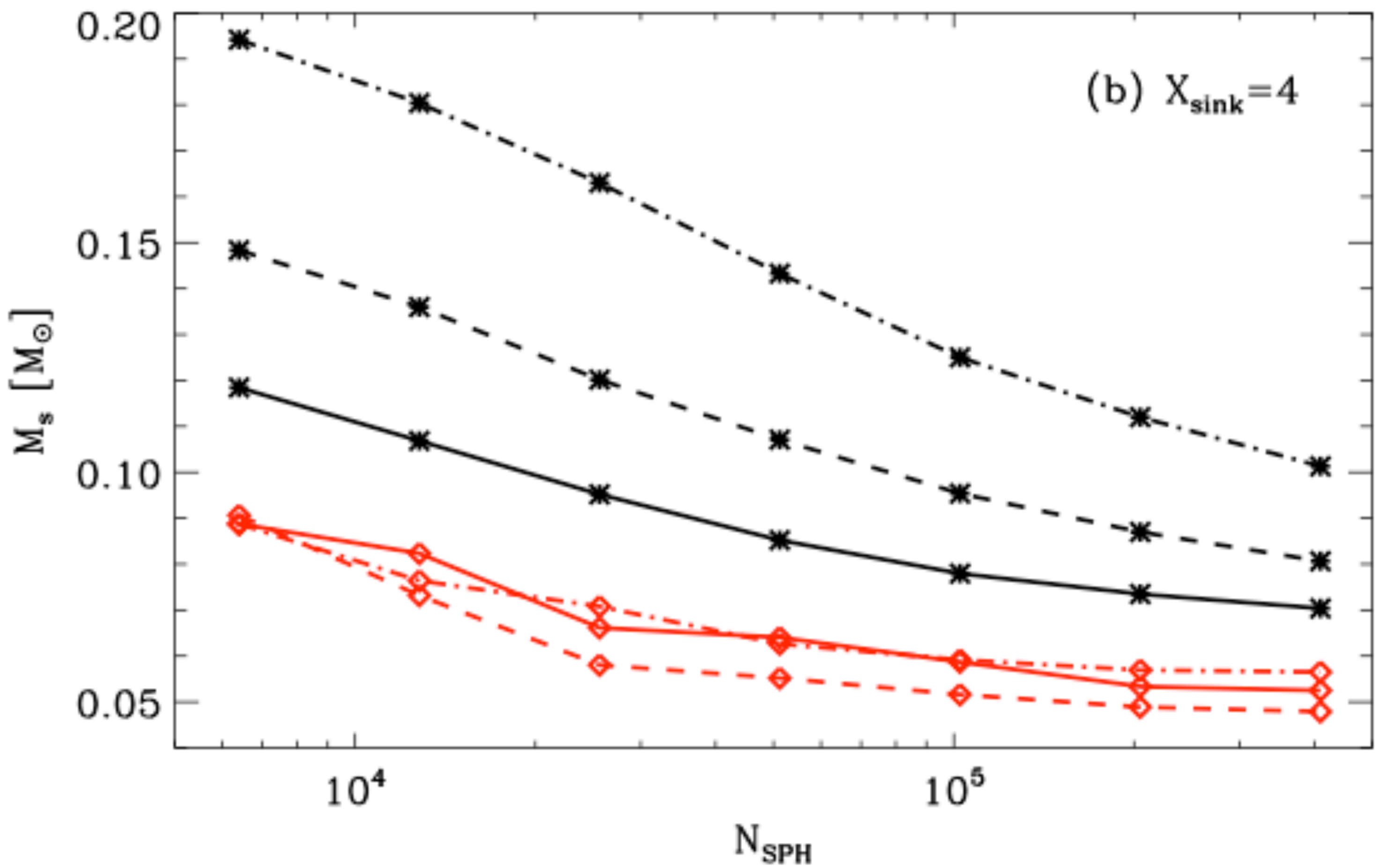


Boss-Bodenheimer test : Convergence of sink properties



- For old sinks, the total mass contained in a sink varies greatly depending on the formation density, and hence the sink radius.
- For new sinks, although the results vary with resolution (external hydrodynamics), they are essentially independent of sink density/radius.

Boss-Bodenheimer test : Convergence of sink properties



- For larger sinks (same formation density), old sinks have even larger masses, but new sinks are still converged at the same masses

Future development

- GANDALF will allow both collisional and collisionless N-body simulations (but far more optimised for collisionless)
- Colliisional N-body will be optimised in the future, particularly with the sub-systems and binary integrators
- Sink particles currently only implemented in SPH schemes
  - Will be added to Meshless scheme soon