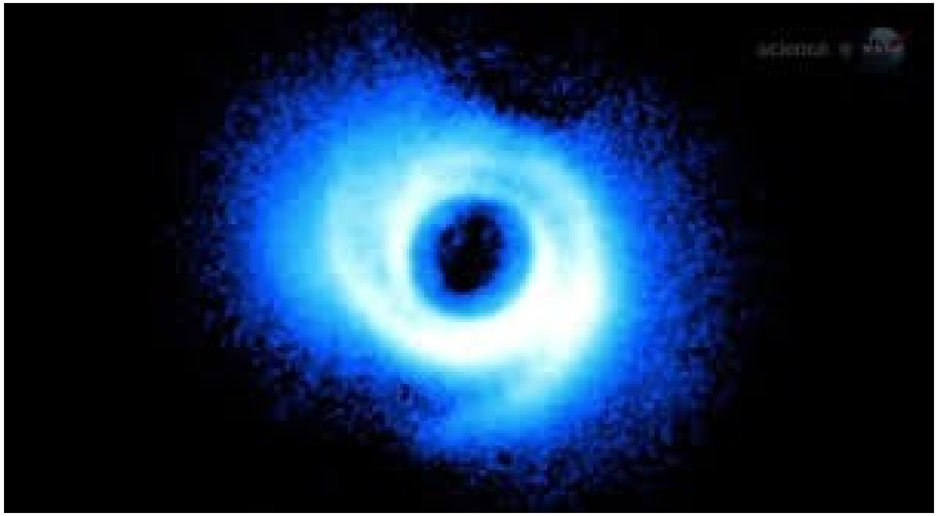
UNIVERSITY OF CAMBRIDGE Institute of Astronomy

Dust dynamics (in GANDALF)

Richard Booth

Why Dust Dynamics?

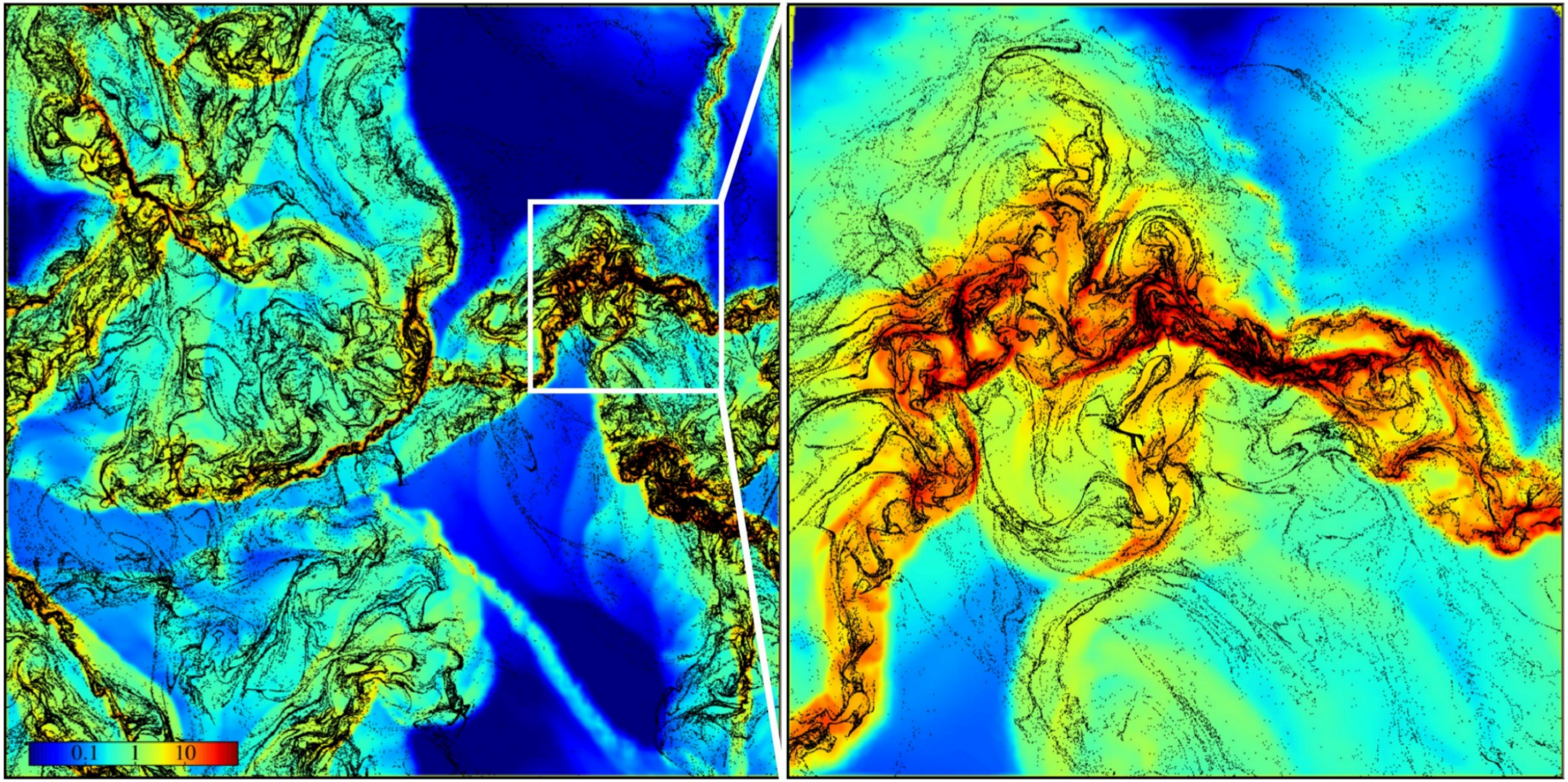


SAO-206462, micron wavelength

HL Tau, mm~ wavelength

Why Dust Dynamics?

Turbulent clouds

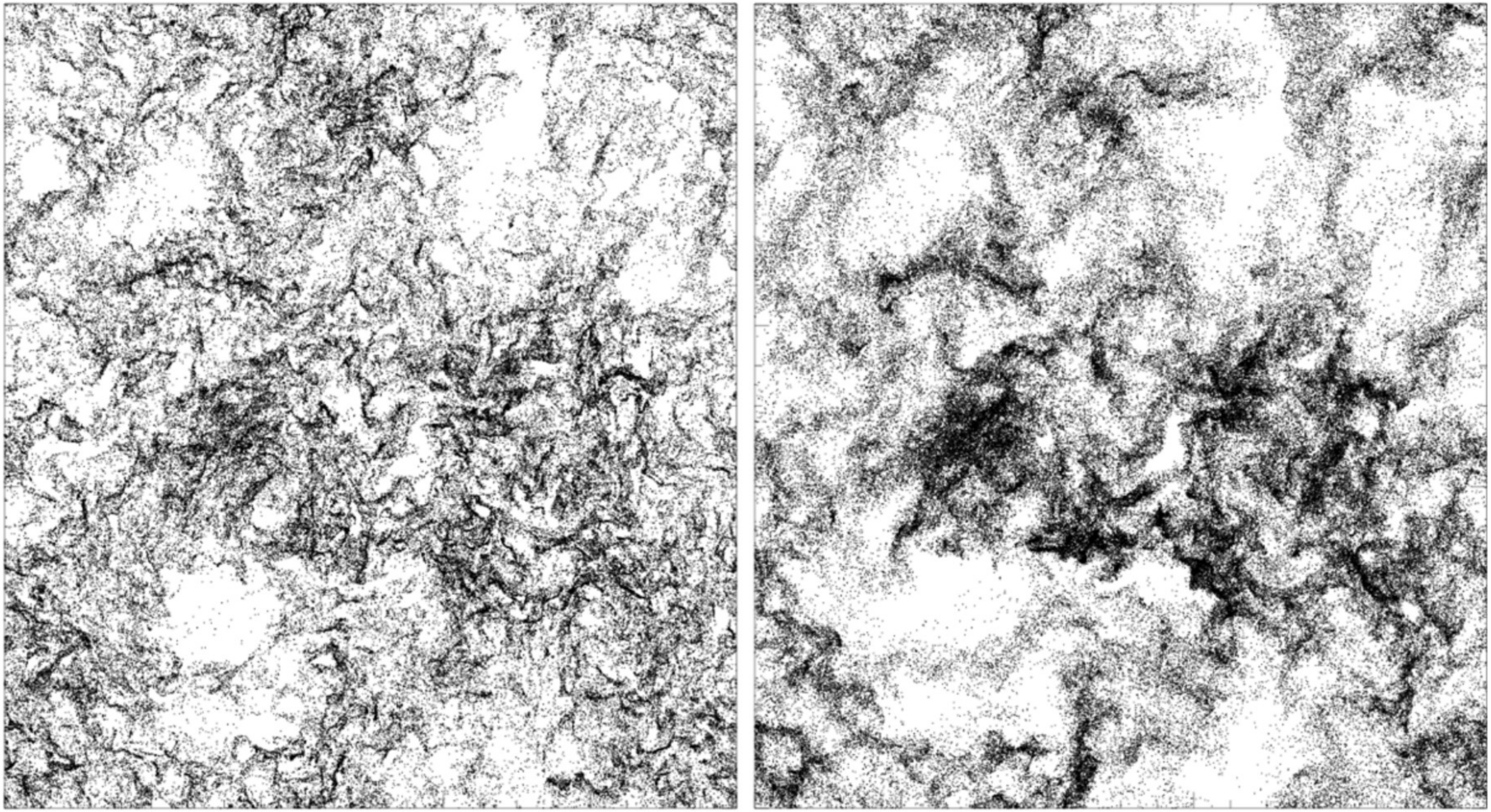


Meshless Finite Volume

Hopkins+ (2015)

Why Dust Dynamics?

Turbulent Clustering



Clustering important for grain growth

Pan & Padoan (2011)

Dust Dynamics:

Key equations

Acceleration of Gas & Dust:

$$\rho_g \frac{D\boldsymbol{v_g}}{Dt} = -\nabla P + \rho_g \boldsymbol{a_g} + \rho_g \rho_d K (\boldsymbol{v_d} - \boldsymbol{v_g})$$

$$m_d \frac{D\boldsymbol{v_d}}{Dt} = m_d \boldsymbol{a_d} - \rho_g K\, m_d (\boldsymbol{v_d} - \boldsymbol{v_g})$$

Epstein Regime:

$$K = \frac{v_T}{\rho_s s}$$

$$t_s = \frac{1}{K(\rho_g + \rho_d)}$$

Stopping time is very short for small particles!

Dust Dynamics:

Equations for centre of mass / relative velocity:

$$\frac{D\,\mathbf{v}_{COM}}{Dt} = \mathbf{a} - F(\Delta\,\mathbf{v}) \qquad \left(\mathbf{a} = \frac{\rho_g\,\mathbf{a_g} + \rho_d\,\mathbf{a_d}}{\rho_d + \rho_g}\right)$$

$$\frac{D\,\Delta\,\mathbf{v}}{Dt} = \frac{-\Delta\,\mathbf{v}}{t_s} + \left(\mathbf{a_d} - \mathbf{a_g} + \nabla\,\frac{P}{\rho_g}\right) + \Delta\,\mathbf{v}\cdot\nabla\,\mathbf{v}_{COM} - G(\Delta\,\mathbf{v})$$

Terms F & G can often be neglected since (see Laibe & Price, 2012... 2014, Youdin & Goodman 2005)

- Depend on density / velocity gradients
- Second order in Delta v

Analytical solution (approx):

$$\Delta\,\mathbf{v}(t+\Delta\,t) = \Delta\,\mathbf{v}(t)\exp(-\Delta\,t/t_s) + \left(\mathbf{a_d} - \mathbf{a_g} + \frac{\nabla\,P}{\rho_g}\right)t_s\left(1 - \exp(-\Delta\,t/ts)\right)$$

$$\mathbf{v}_{COM}(t+\Delta\,t) = \mathbf{v}_{COM}(t) + \mathbf{a}\,\Delta\,t$$

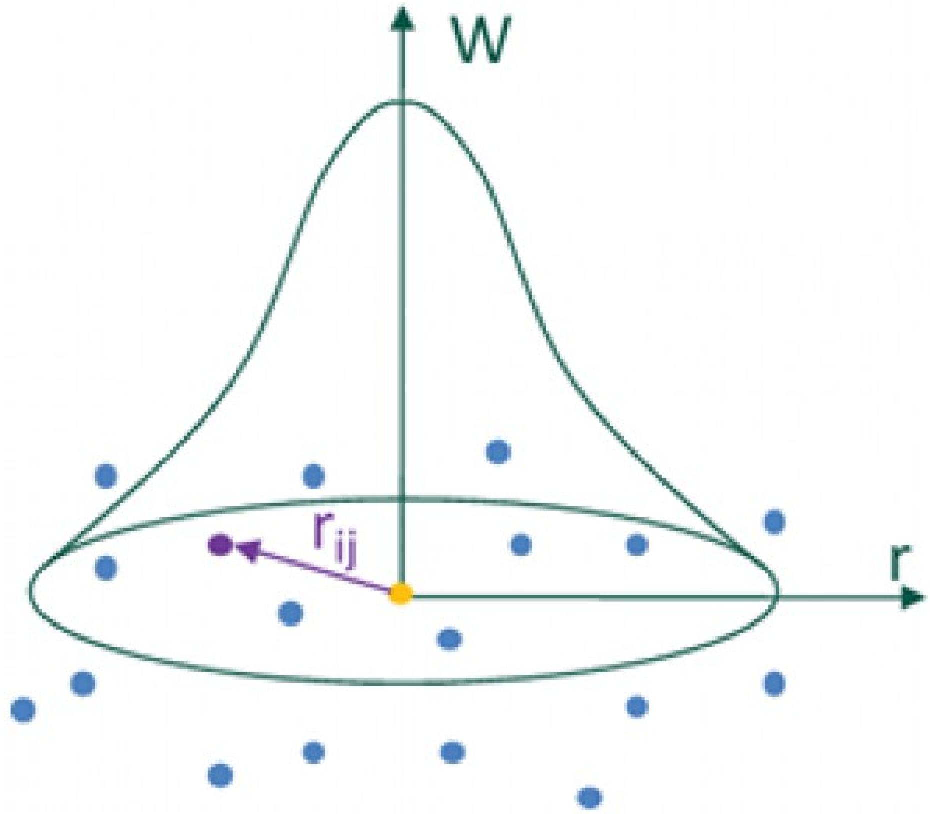
(Constant accelerations and stopping time)

Interpolation:

Direct

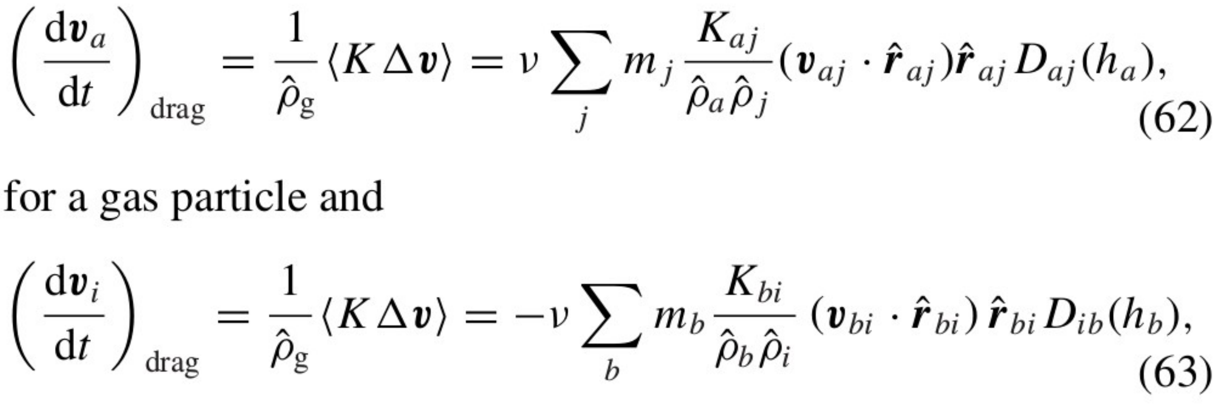
$$\langle \rho \rangle = \Sigma_i m_i W(r_{ij}, h_j)$$

$$\langle \boldsymbol{v} \rangle = \Sigma_i \boldsymbol{v}_{\boldsymbol{i}} \frac{m_i}{\rho_i} W(r_{ij}, h_j)$$



Drag force not aligned with particle separation:

- Angular momentum not conserved
- Use projected forces?



$$\left(\frac{\mathrm{d}\boldsymbol{v}_a}{\mathrm{d}t}\right)_{\mathrm{drag}} = \frac{1}{\hat{\rho}_{\mathrm{g}}}\langle K\,\Delta\boldsymbol{v}\rangle = \nu \sum_j m_j \frac{K_{aj}}{\hat{\rho}_a \hat{\rho}_j}(\boldsymbol{v}_{aj}\cdot\hat{\boldsymbol{r}}_{aj})\hat{\boldsymbol{r}}_{aj} D_{aj}(h_a), \tag{62}$$

for a gas particle and

$$\left(\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t}\right)_{\mathrm{drag}} = \frac{1}{\hat{\rho}_{\mathrm{g}}}\langle K\,\Delta\boldsymbol{v}\rangle = -\nu \sum_b m_b \frac{K_{bi}}{\hat{\rho}_b \hat{\rho}_i}(\boldsymbol{v}_{bi}\cdot\hat{\boldsymbol{r}}_{bi})\,\hat{\boldsymbol{r}}_{bi} D_{ib}(h_b), \tag{63}$$

Laibe & Price (2012)

Interpolation:

Projected – Time integration

$$\mathbf{v}_{\mathrm{D}}^{i}(t+\delta t, \mathbf{r}_i) = \tilde{\mathbf{v}}_{\mathrm{D}}^{i}(t+\delta t, \mathbf{r}_i) - \frac{\nu}{N_i} \sum_{k}^{\mathrm{Gas}} \frac{m_k}{\rho_k} \left(\mathbf{S}_{ik} \cdot \hat{\mathbf{r}}_{ik}\right) \hat{\mathbf{r}}_{ik} W(|\mathbf{r}_{ik}|, h_k), \qquad (27)$$

Zero drag case
+ drag correction

$$\mathbf{v}_{\mathrm{G}}^{j}(t+\delta t, \mathbf{r}_j) = \tilde{\mathbf{v}}_{\mathrm{G}}^{j}(t+\delta t, \mathbf{r}_j) + \nu \sum_{k}^{\mathrm{Dust}} \frac{m_k}{N_k \rho_j} (\mathbf{S}_{kj} \cdot \hat{\mathbf{r}}_{kj}) \hat{\mathbf{r}}_{kj} W(|\mathbf{r}_{kj}|, h_j), \qquad (28)$$

$$u_{\mathrm{G}}^{j}(t+\delta t, \mathbf{r}_j) = \tilde{u}_{\mathrm{G}}^{j}(t+\delta t, \mathbf{r}_j) + \sum_{k}^{\mathrm{Dust}} \frac{m_k}{N_k \rho_k} \Big[\left(\mathbf{S}_{kj} \cdot \hat{\mathbf{r}}_{kj}\right) \left(\mathbf{v}_{kj} \cdot \hat{\mathbf{r}}_{kj}\right) W(|\mathbf{r}_{kj}|, h_j) - \frac{1}{2} \left(1 + \rho_k/\rho_j\right) \left(\mathbf{S}_{kj} \cdot \hat{\mathbf{r}}_{kj}\right)^2 W(|\mathbf{r}_{kj}|, h_j) \Big] \qquad (29)$$

Loren-Aguilar & Bate (2015)

Interpolation:

Projected – Time integration

$$\mathbf{v}_{\mathrm{D}}^{i}(t+\delta t,\mathbf{r}_i) = \tilde{\mathbf{v}}_{\mathrm{D}}^{i}(t+\delta t,\mathbf{r}_i) - \frac{\nu}{N_i}\sum_{k}^{\mathrm{Gas}} \frac{m_k}{\rho_k}\left(\mathbf{S}_{ik}\cdot\hat{\mathbf{r}}_{ik}\right)\hat{\mathbf{r}}_{ik} W(|\mathbf{r}_{ik}|,h_k), \qquad (27)$$

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$$\boldsymbol{S} = \Delta\boldsymbol{v}\,\zeta(\Delta t) - \Delta\boldsymbol{a}\,\Lambda(\Delta t)$$

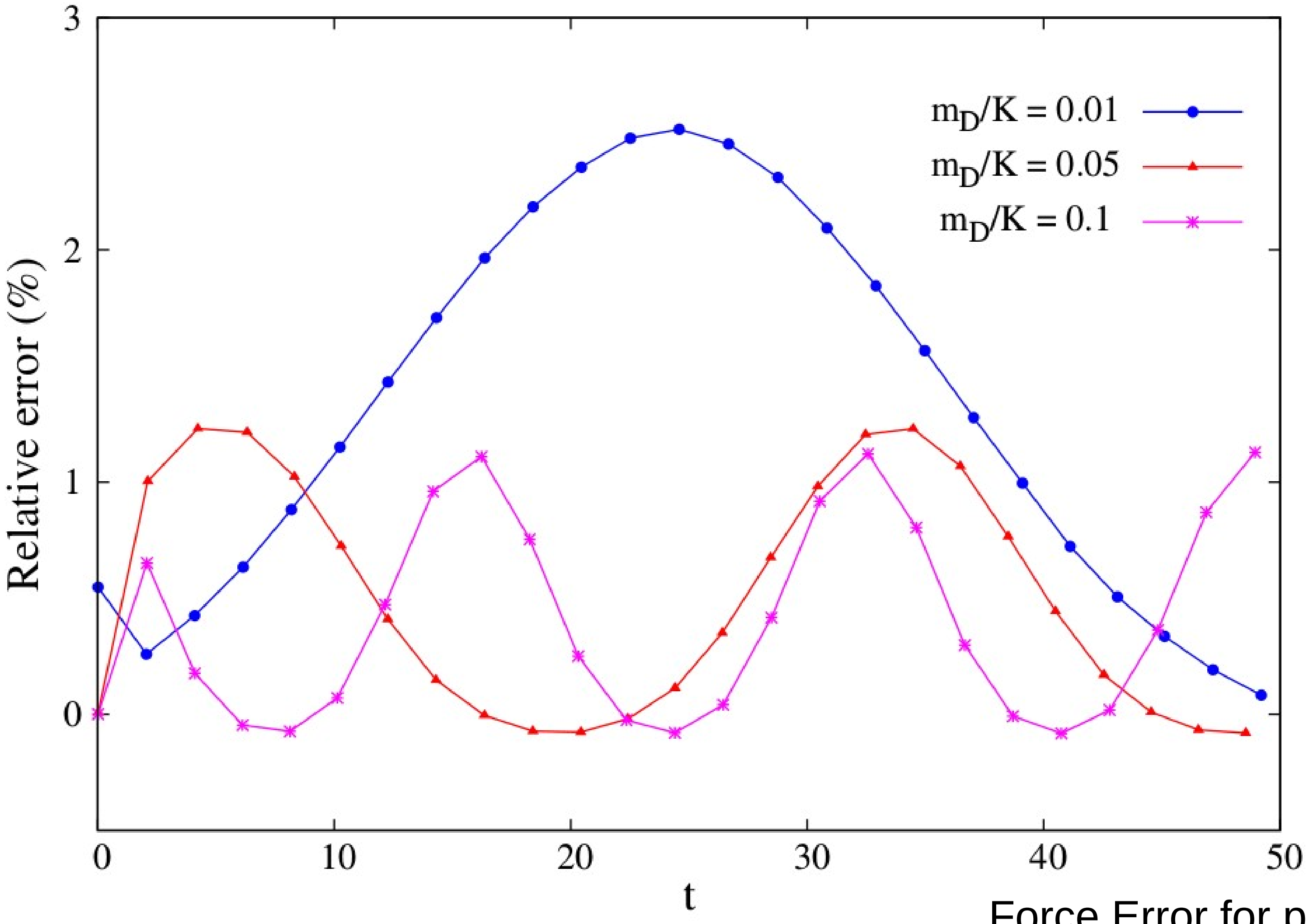
$$\zeta(\Delta t) = \frac{1-\exp(-\Delta t/t_s)}{1+\epsilon} \qquad \epsilon = \frac{\rho_d}{\rho_g}$$

$$\Lambda(\Delta t) = (\Delta t + t_s)\,\zeta(\Delta t) - \frac{\Delta t}{1+\epsilon}$$

Interpolation:

Projected Force Error

Loren-Aguilar & Bate (2015)

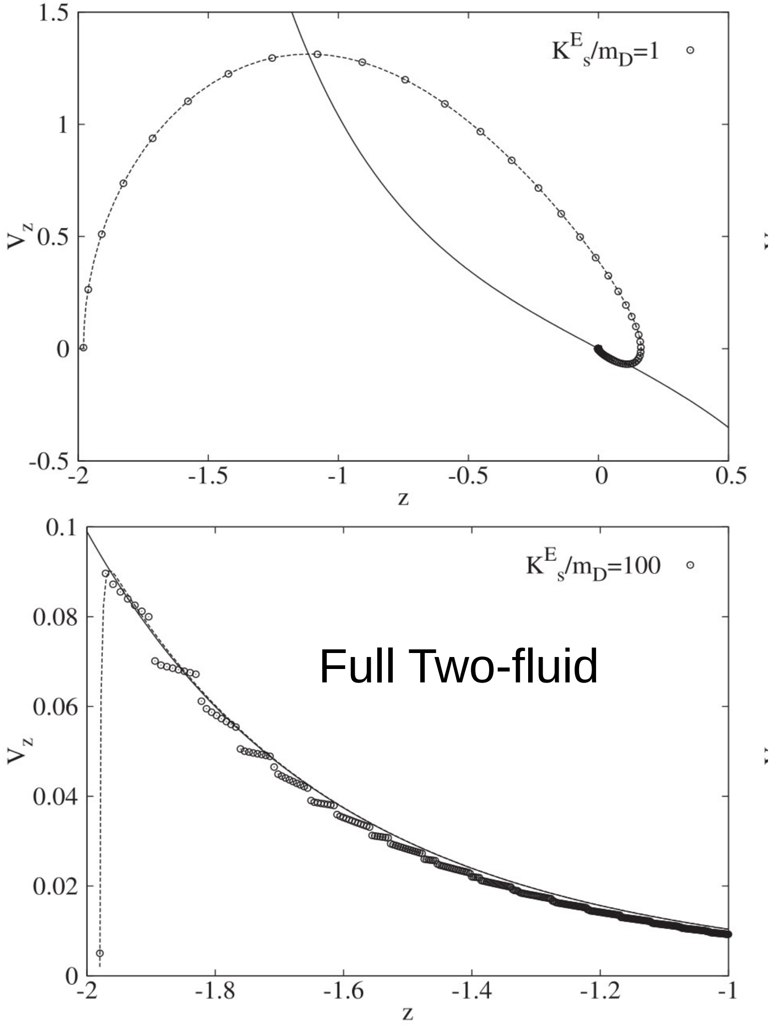


Force Error for projected forces:

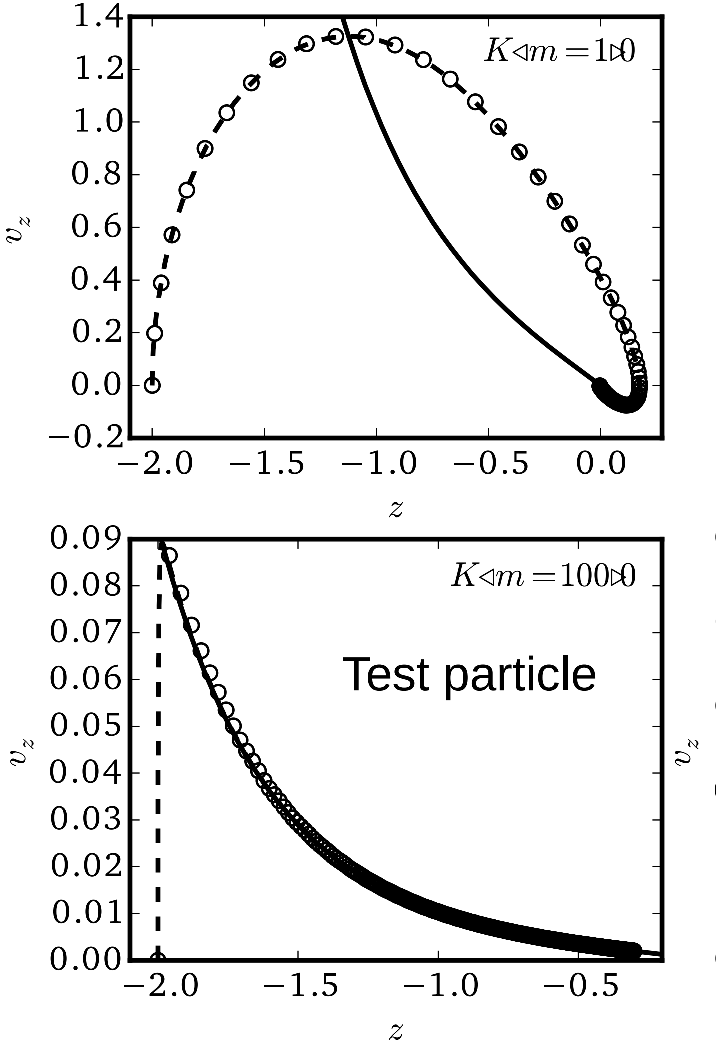
- Uniform density

Interpolation:

Settling Test



Full Two-fluid



Test particle

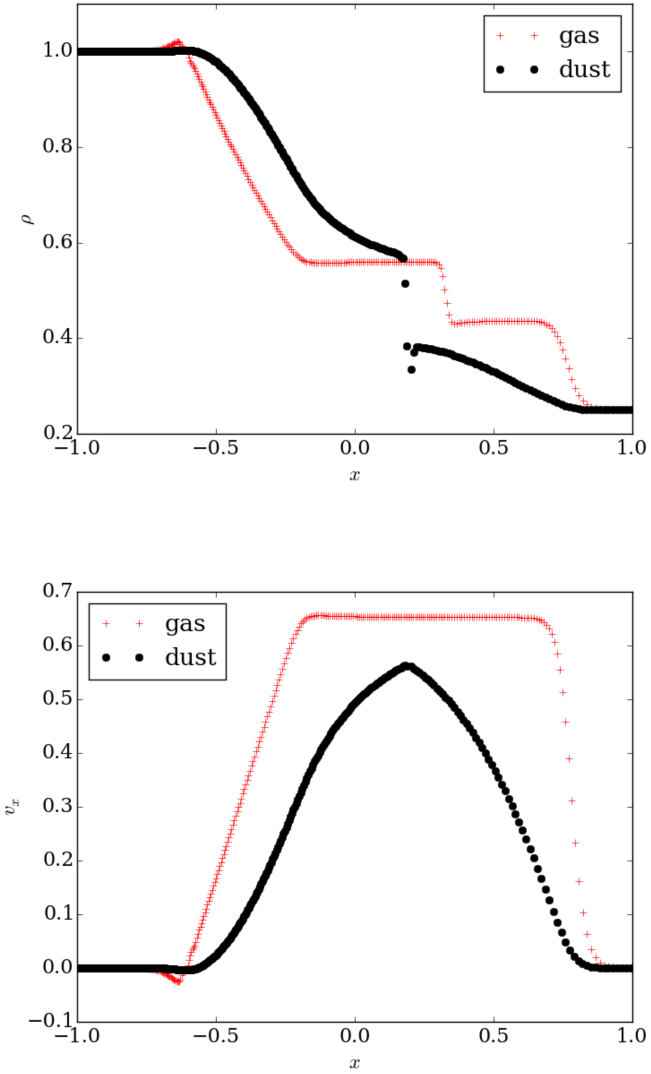
Loren-Aguilar+ (2014)

Booth, Sijacki & Clarke (2015)

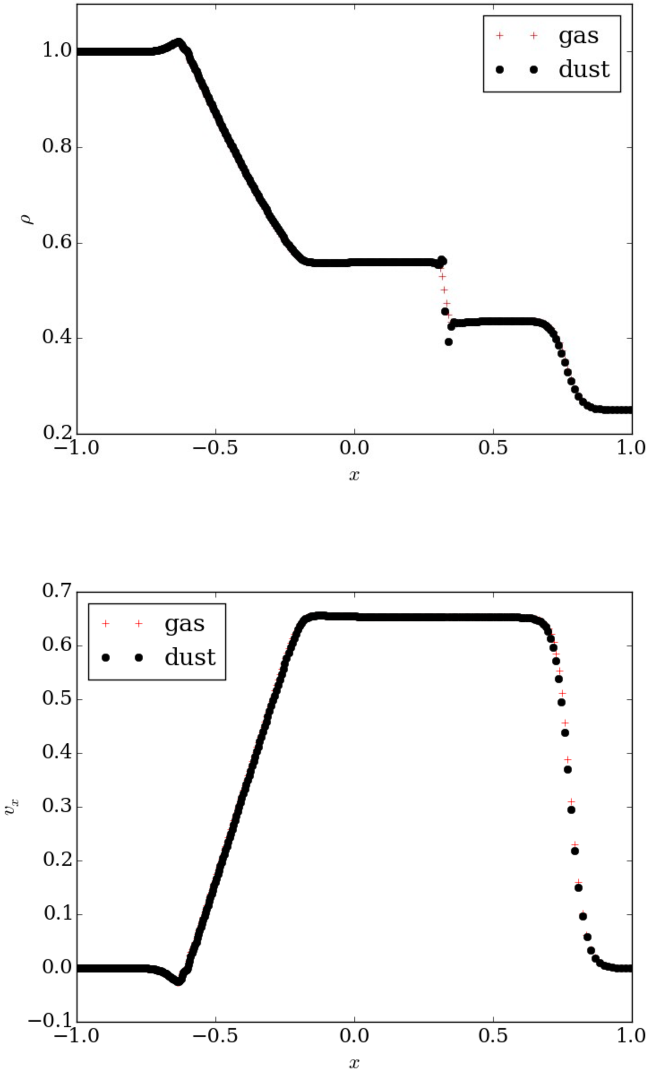
Current Status in GANDALF:

Test Particle limit only:

$t_s = 0.25, t = 0.75$

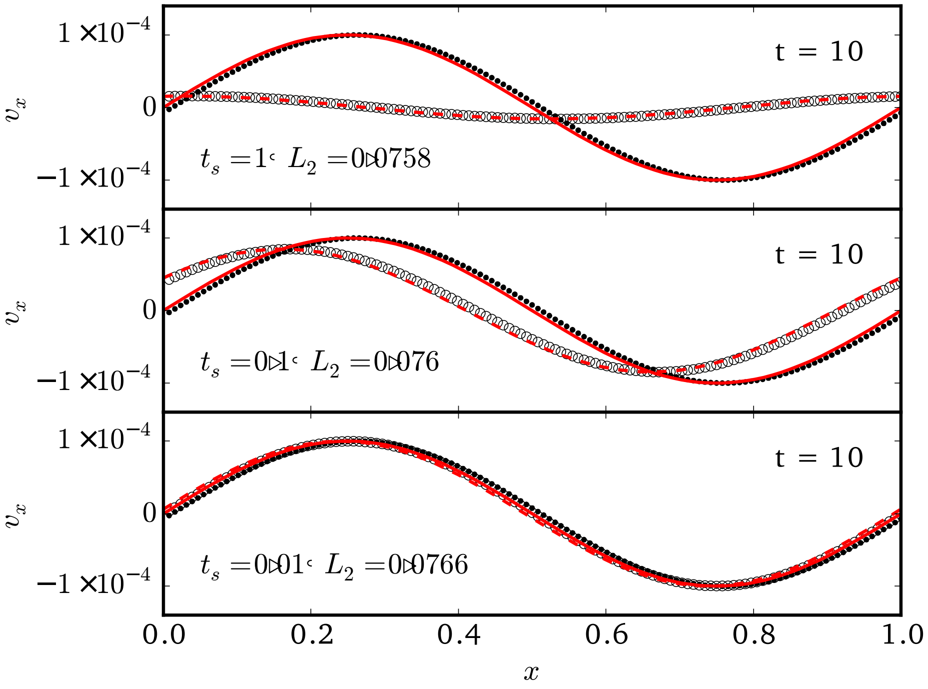


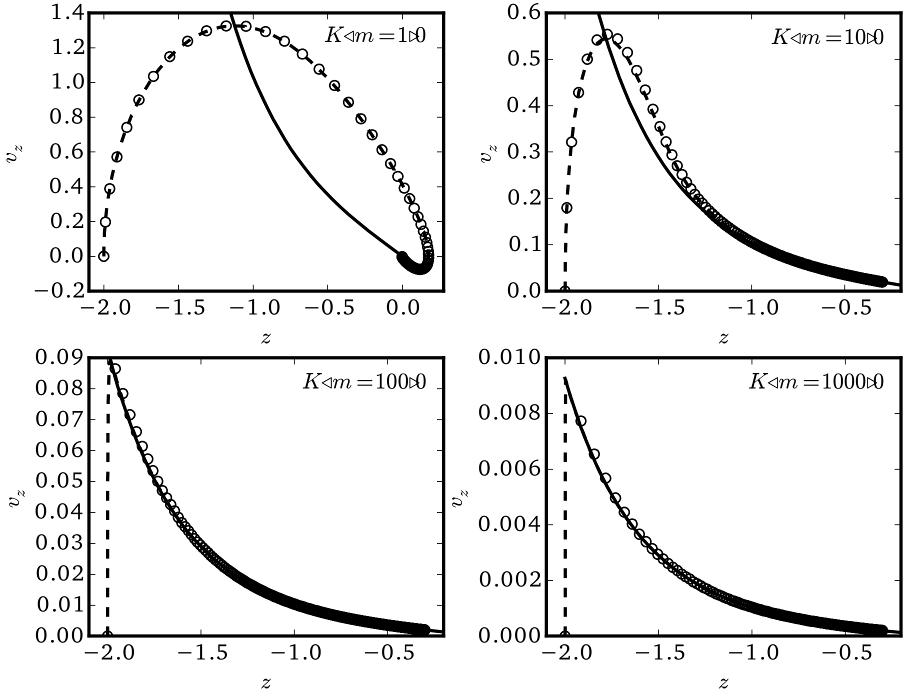
$t_s = 0.0025, t = 0.75$



Tests:

Standard tests are easy





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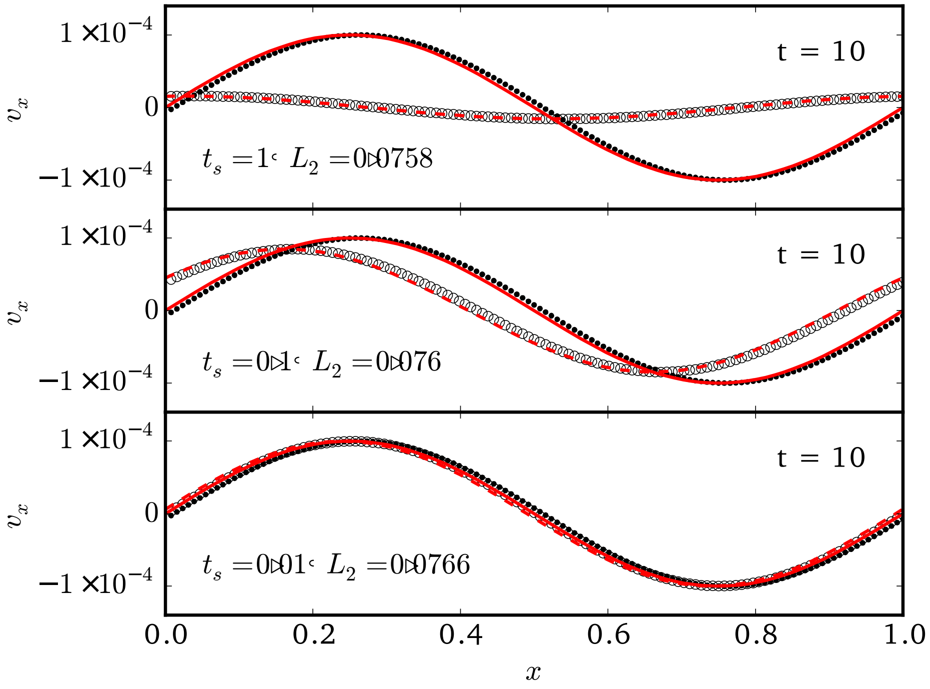
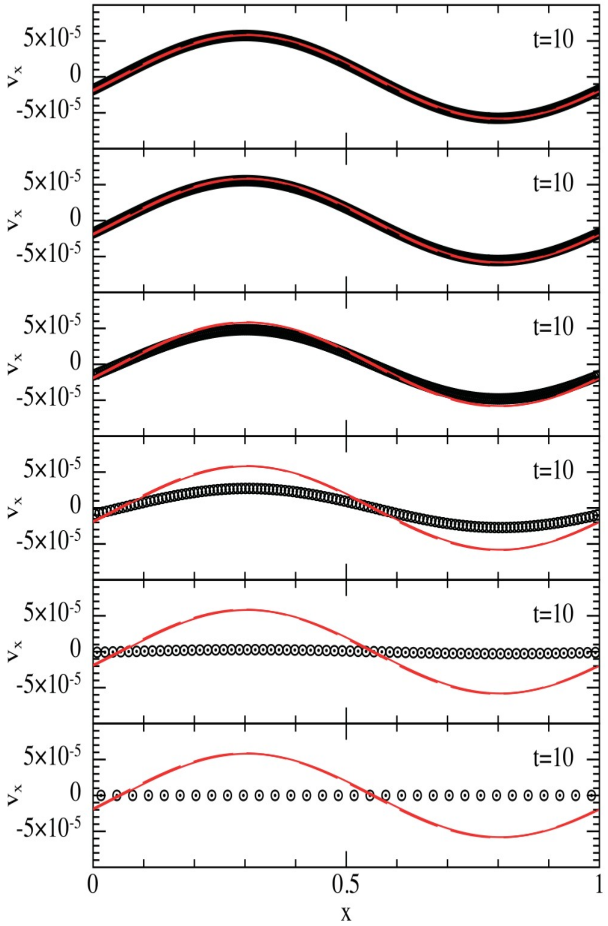
Tests:

Standard tests are easy (in the test particle limit)

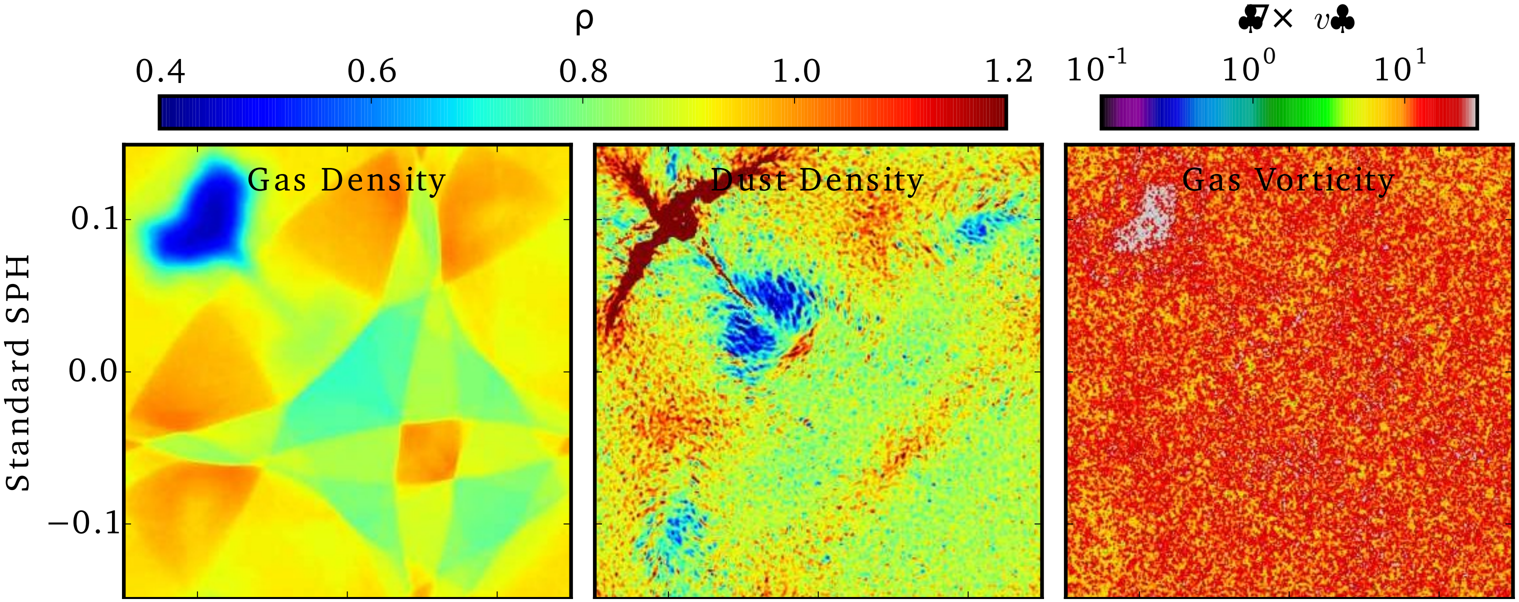




Full two-fluid equations are dissipative though
  - Low resolution will over-damp waves
  - Semi-implicit approach helps
  - Damping weaker in when dust-to-gas ratio is low

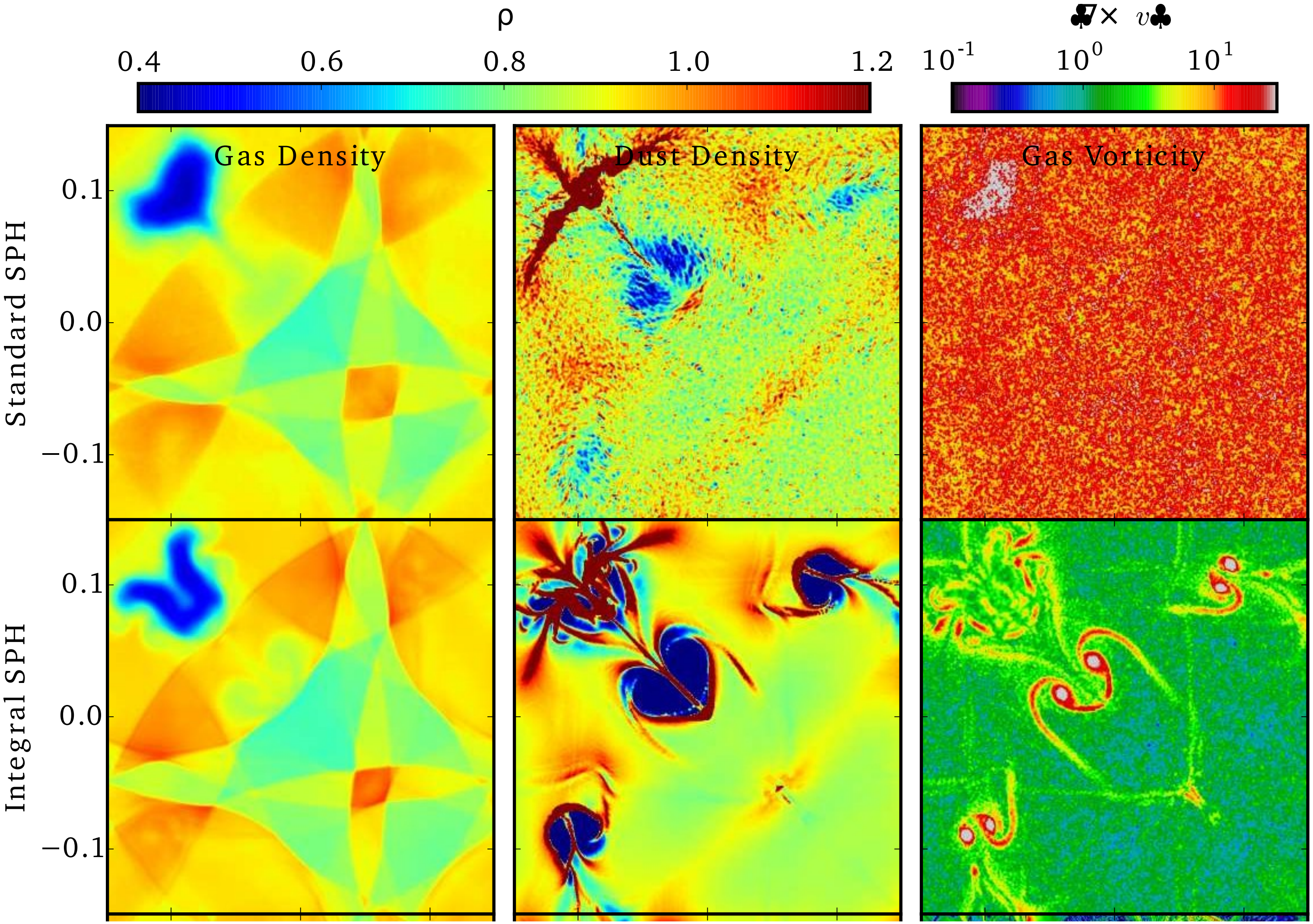
Tests:

Multi-dimensional



Tests:

Multi-dimensional



Shear Test:

Shu, Milione, Roberts (1973)

- Isothermal gas
- Impose a background potential
  - Flat rotation curve:

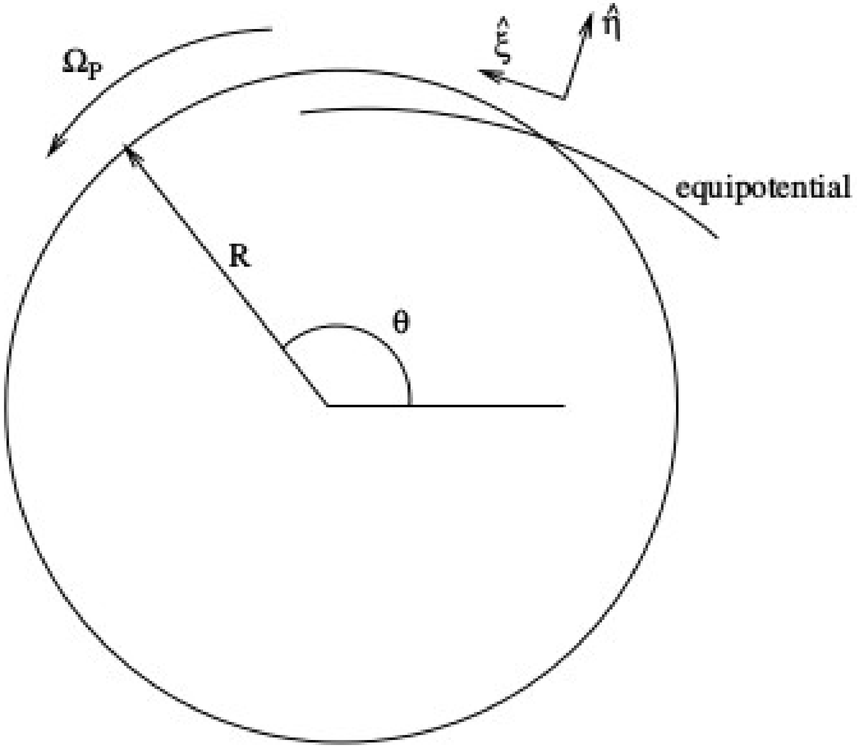
$$v_0(R) = v_{max}\sqrt{F_b \epsilon_b \exp(-\epsilon_b R) + 1 - \exp(\epsilon_d R)}$$

  - Logarithmic spiral perturbation ϵ_b

$$V_s = A_0 R \exp(-\epsilon_s R) \cos(\chi)$$

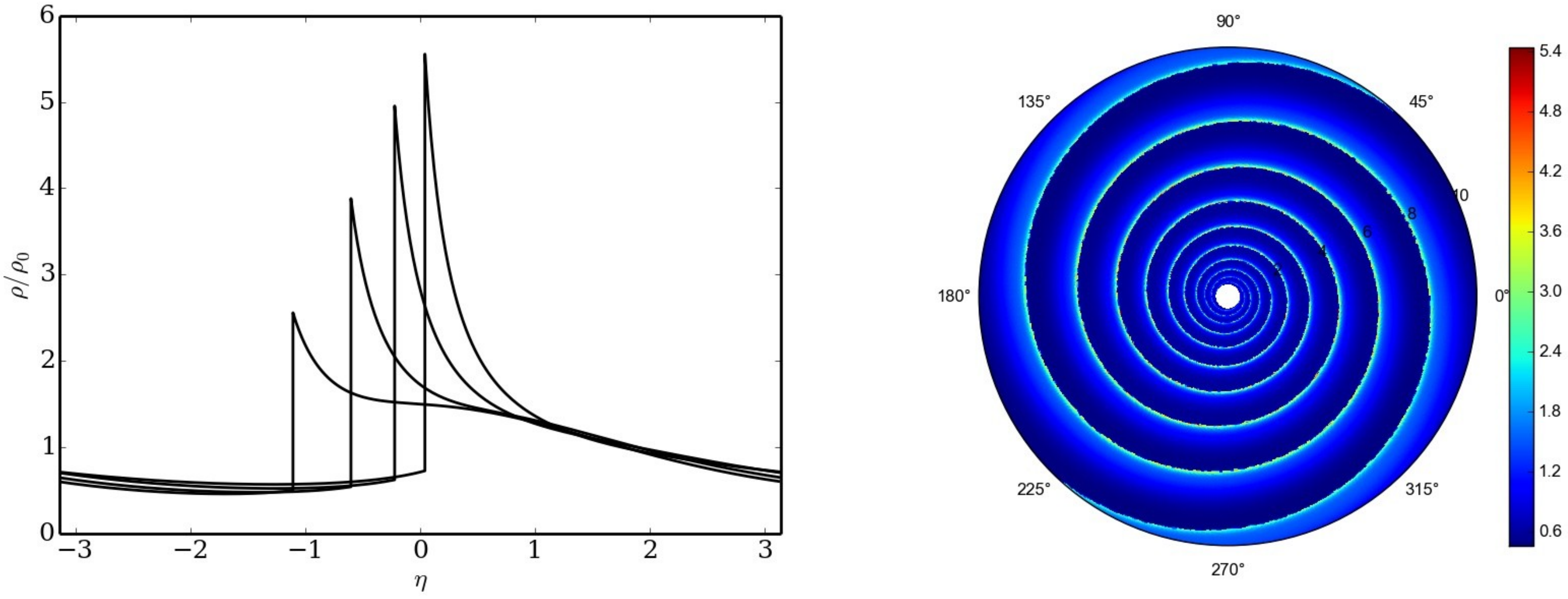
$$\chi = -\frac{m}{\tan i} \ln(R) - m(\theta - \Omega_p t)$$

  - Pattern speed, Ω_p
- Find solutions along stream lines



Gittins & Clarke (2004)

Shear Test:
Stream-line solution

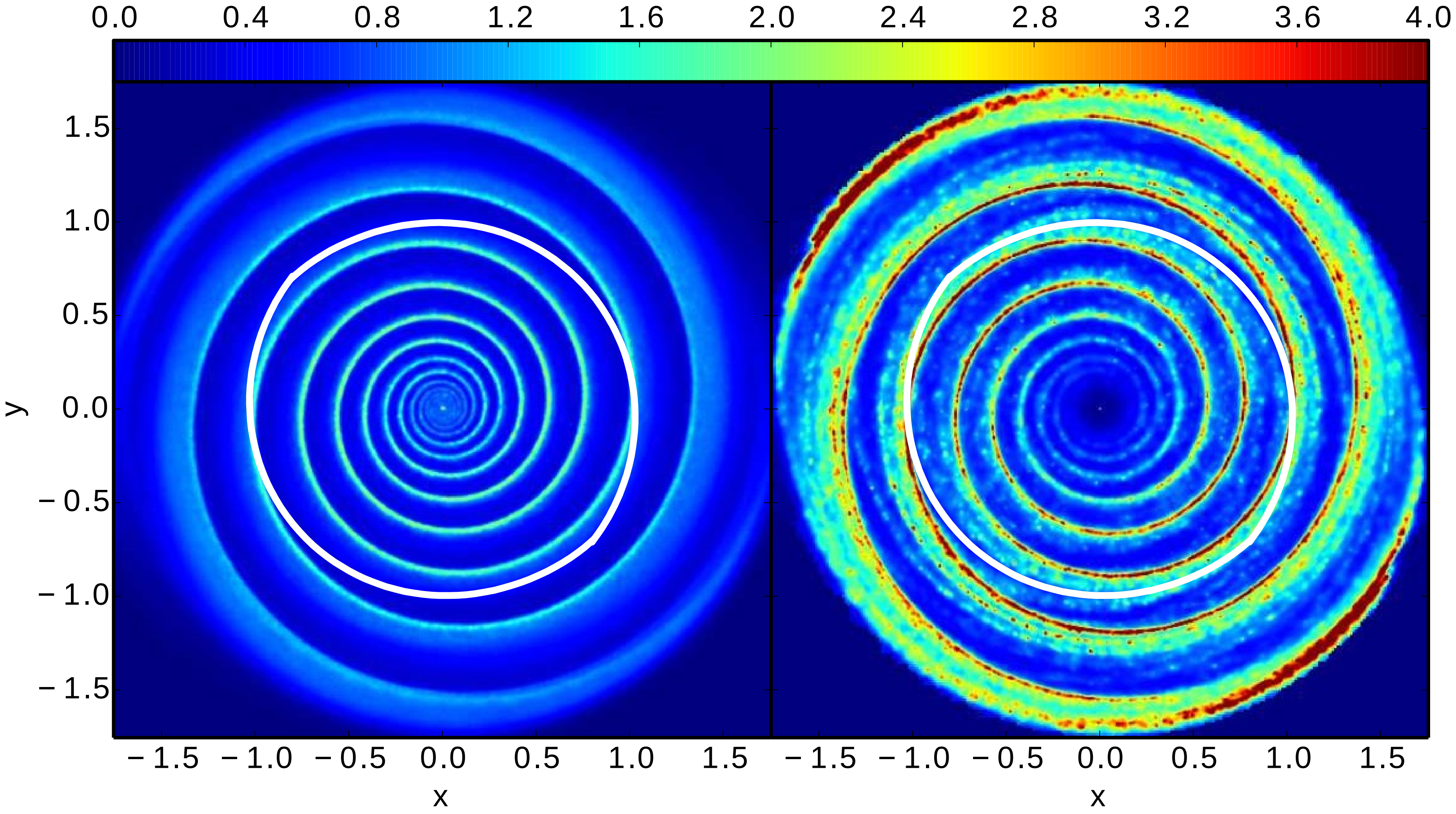


Stream-lines used to build the full disc structure

Modelling:

Test Problems

(Booth+ 2015)



Modelling:

Test Problems

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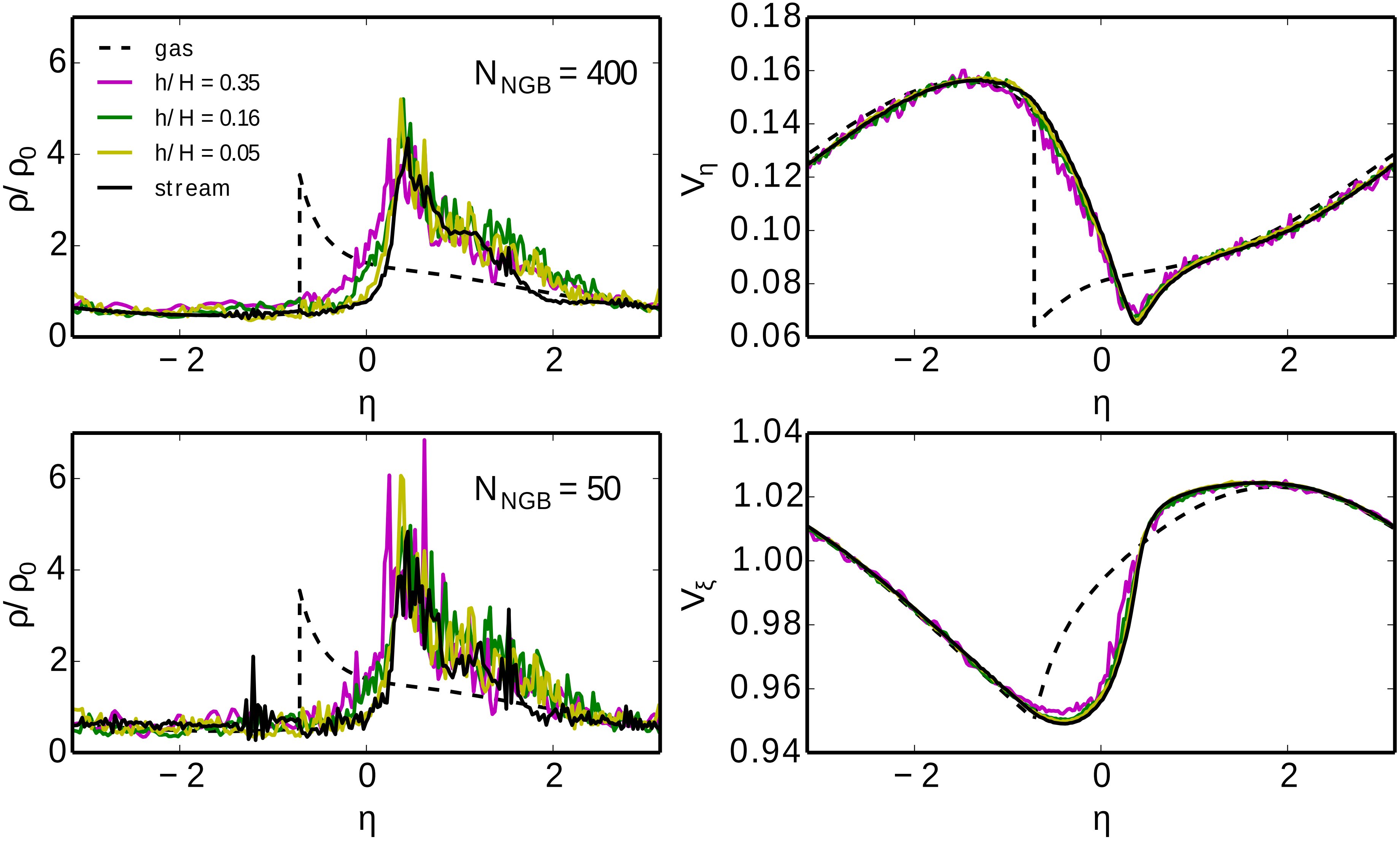
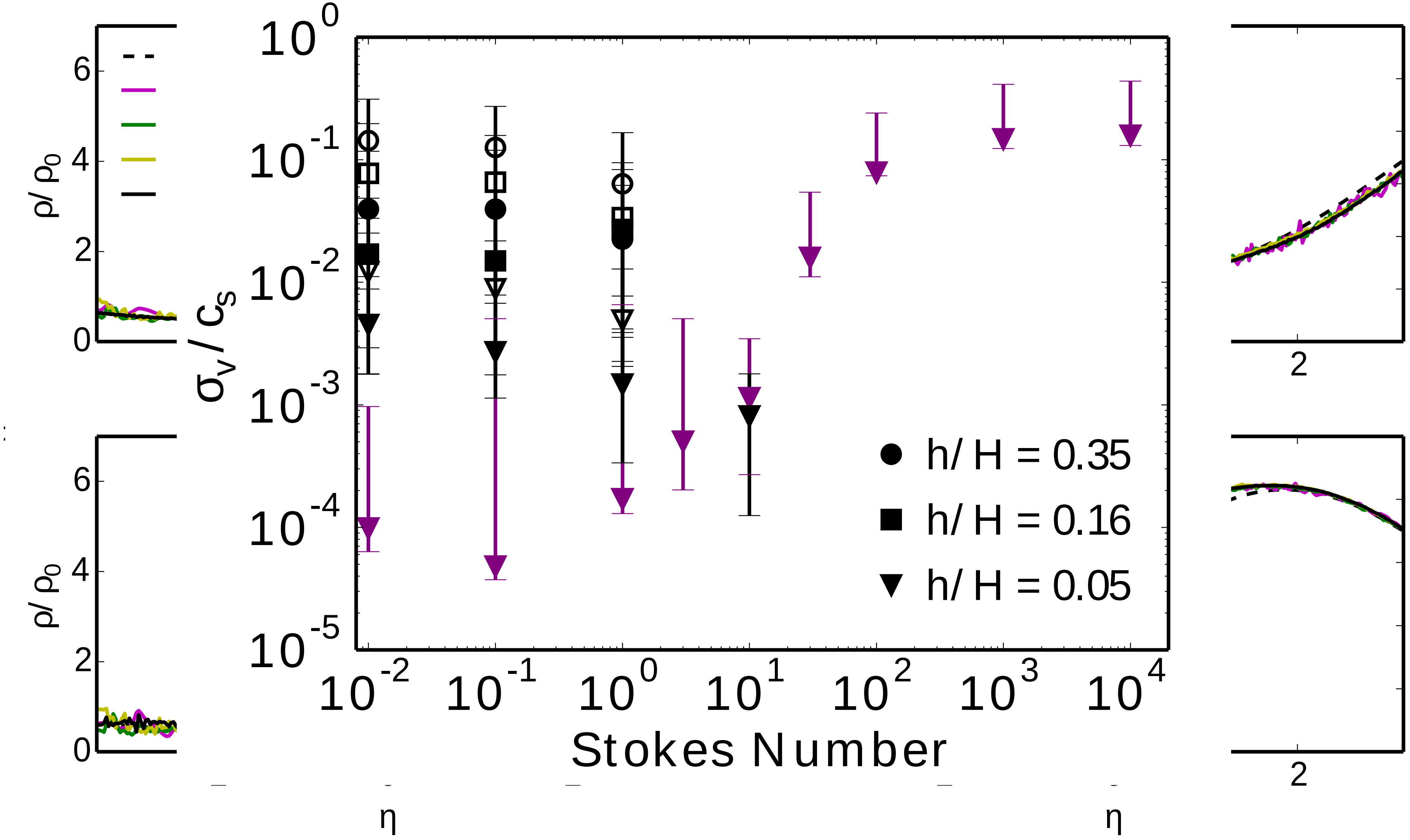


Modelling:

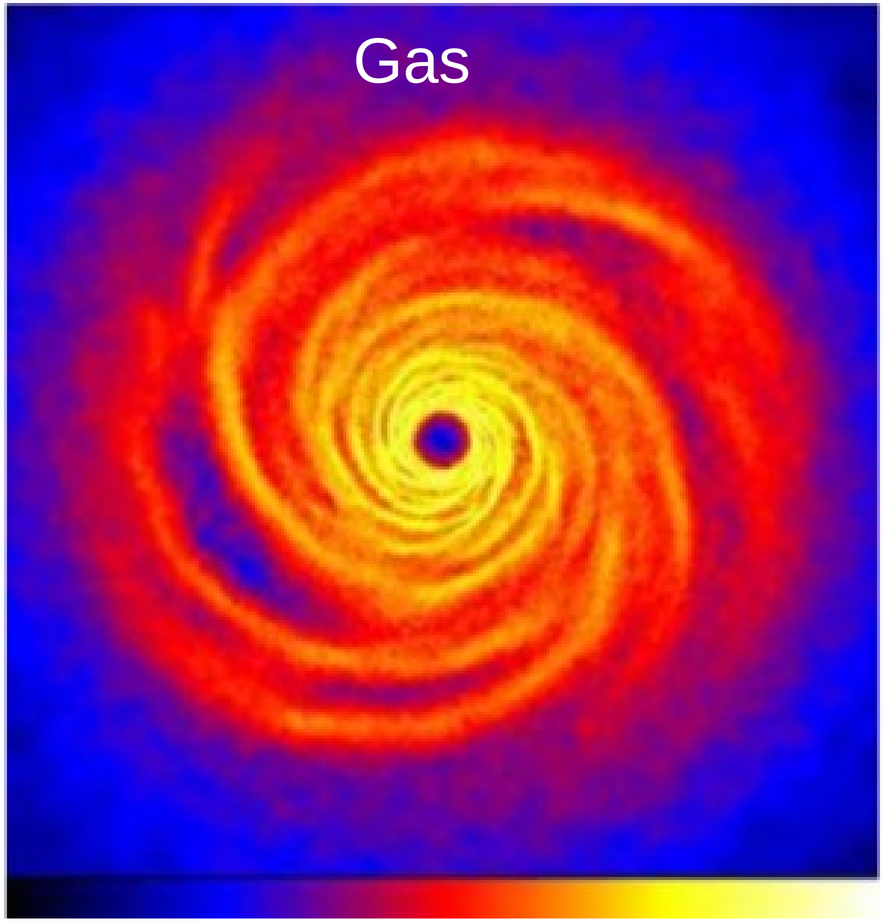
Test Problems

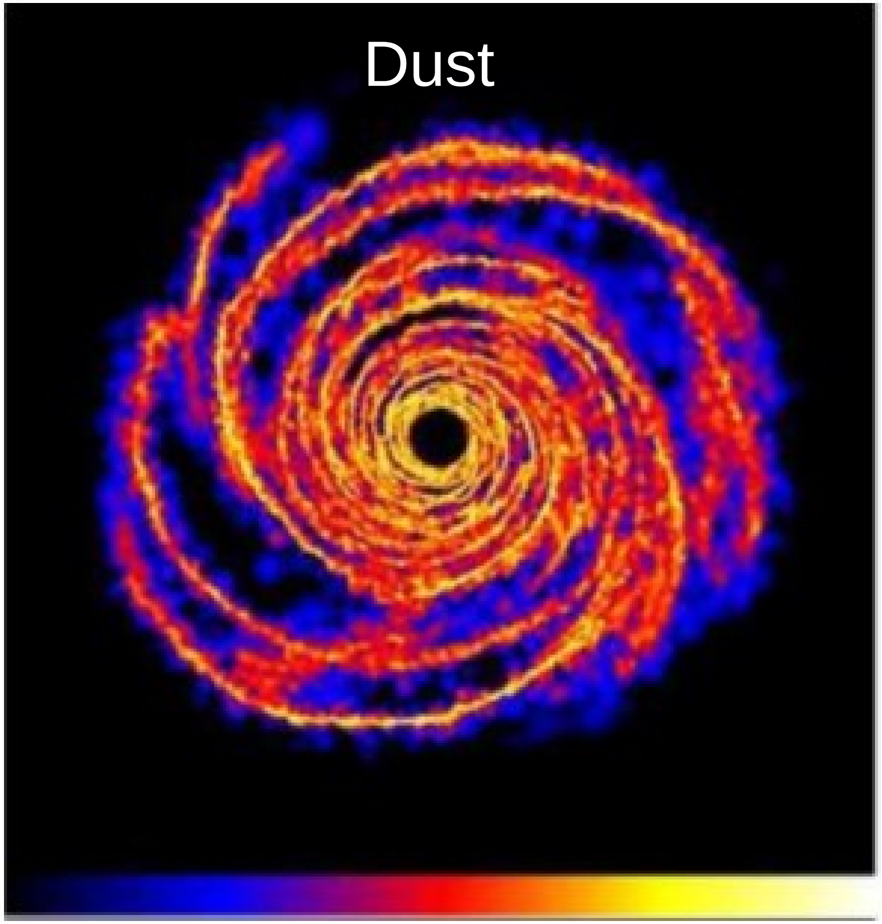
(Booth+ 2015)



Application:

Dust in self-gravitating protoplanetary discs

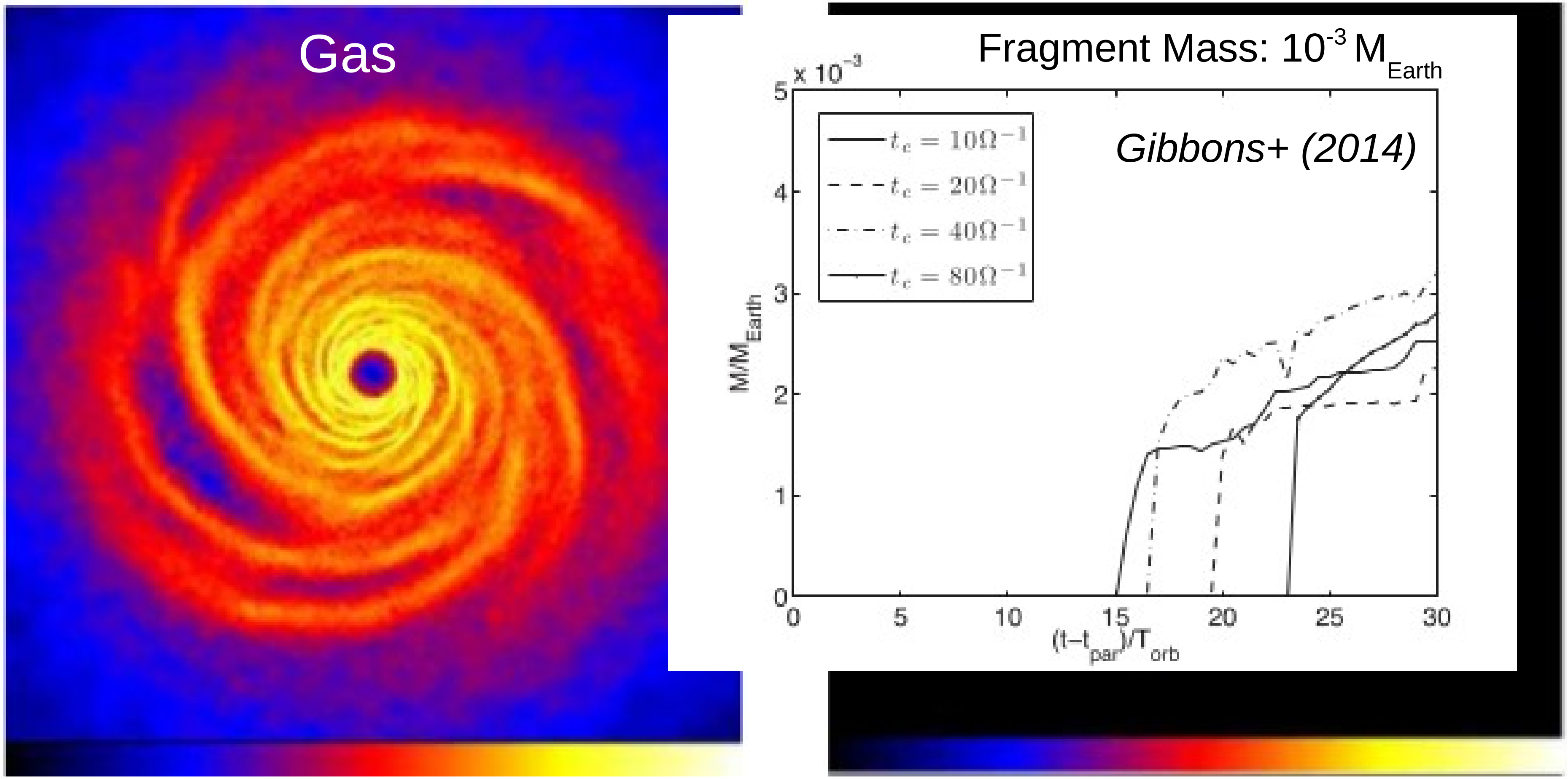




Rice+ (2004)

Application:

Dust in self-gravitating protoplanetary discs



Rice+ (2004)

Key Questions:

Growth & Fragmentation

- Most likely place for this to happen: Class 1 discs
  - mm/cm grains seen(Miotello+ 2014)
  - Q ~ 1
  - requires St ~ 0.01 to 0.1 at 30au
  - **Is this consistent with self-gravity?**

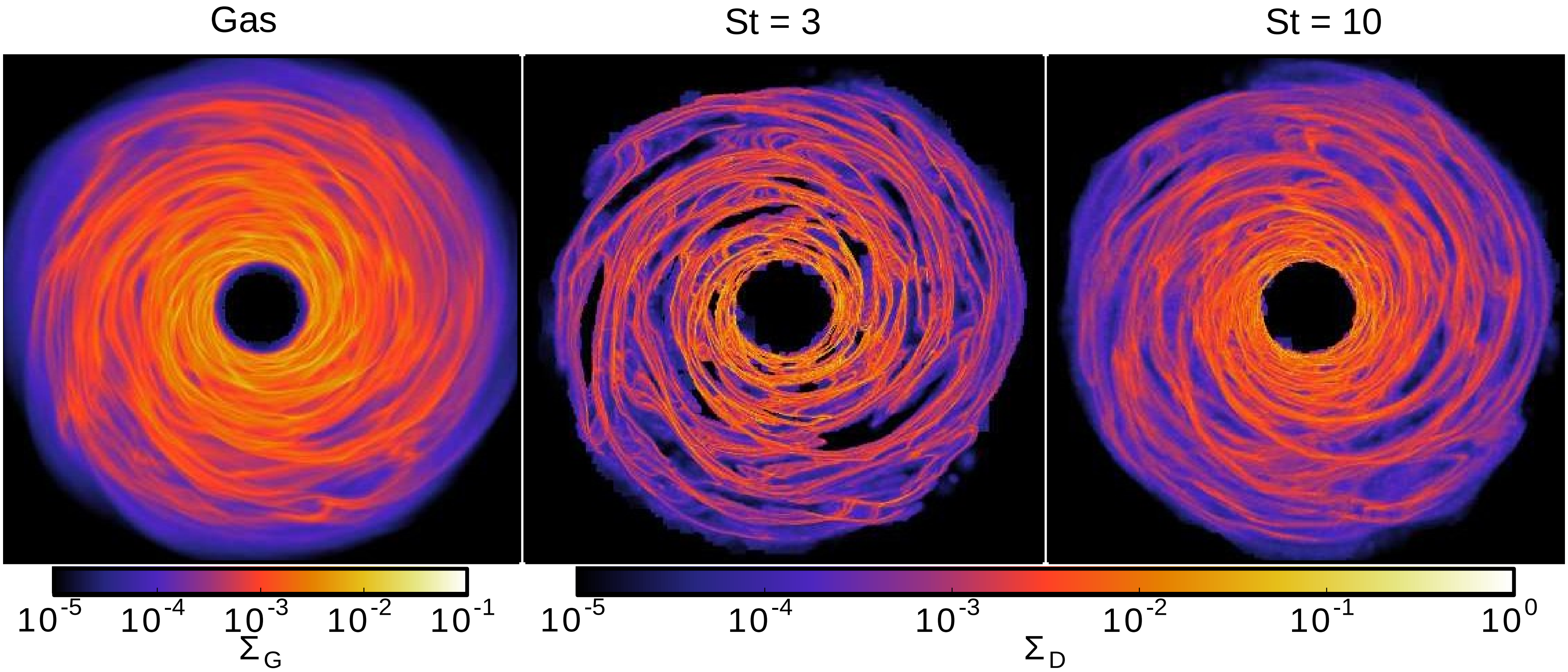
- **How large do grains need to be for trapping to be effective enough?**

- **Do collisions lead to fragmentation?** Can growth to St ~ 1 occur?
  - St = 1 corresponds to few 10 cm (high density)
  - Fragmentation velocity
    - ~ 1 m/s for silicates (Guttler+ 2010)
    - few 10 m/s for ices (Wada+ 2009; Gundlach & Blum, 2015)

- Large velocity dispersion for planetesimals
  - Of order c_s: ~ few 100 m/s (Walmswell+ 2013)
  - **How much does coupling reduce this?**

Modelling:

Simulations (Dimensionless!)



2D Simulations:
- Easier to reduce noise
- Need to resolve scales < H for St < 1
- 1, 4 & 16 million particles per phase

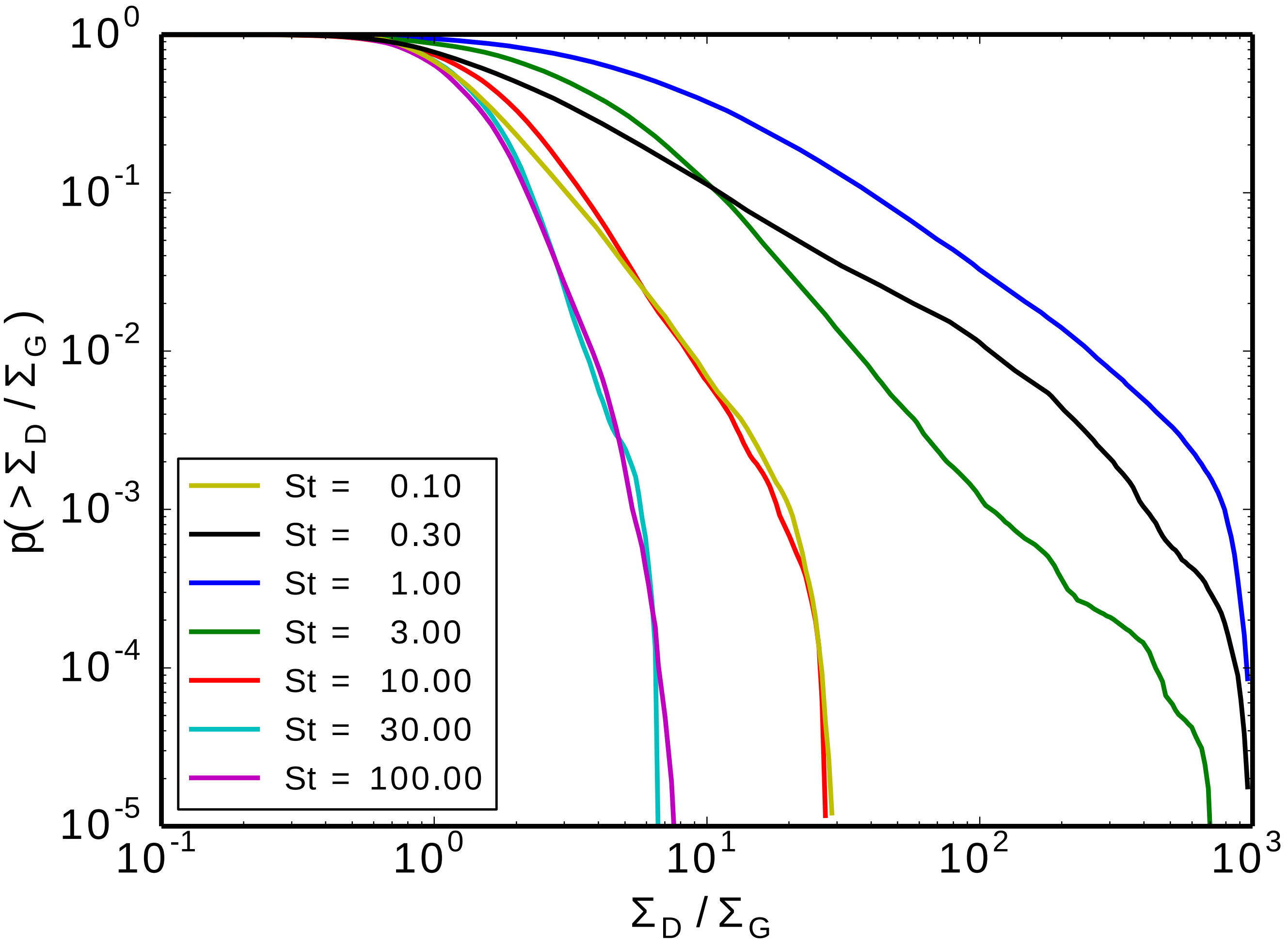
Beta Cooling: $t_c = \beta\,\Omega^{-1}$ ($\beta = 10$)

Fixed Stokes number: $t_s = St\,\Omega^{-1}$

Test particle limit

Disc mass = 0.1 Star mass

Density enhancement



Fraction of particles in high density regions:

- Density enhancement > 100 for 0.3 < St < 3
  - Gravitational collapse needs St > 0.3

Relative velocities:

Equal sized particles

- Measure distribution of relative velocities,

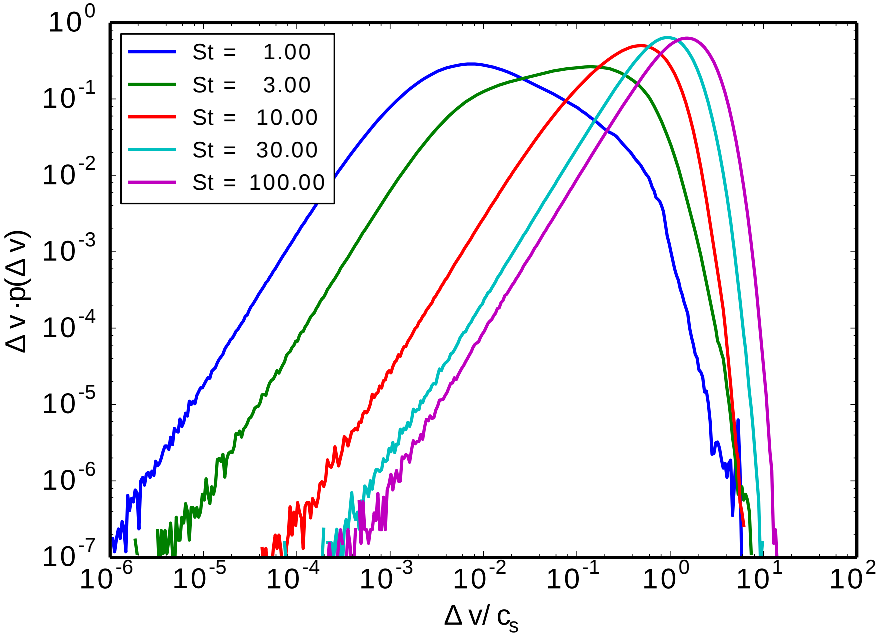
$$P(\Delta v)$$

  - Using r.m.s relative velocity

$$\Delta v = |\mathbf{\Delta v}|$$

  - Rate of collisions:

$$\Gamma \propto \Delta v P(\Delta v)$$



- Intermediate regime for

  St < 3

Relative velocities:

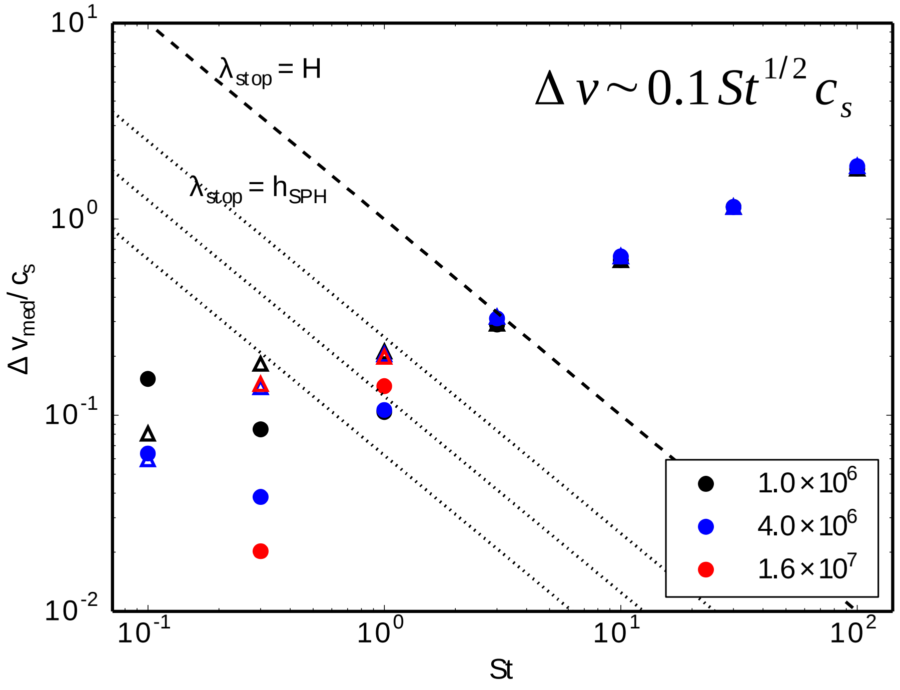
Equal Sized Particles

$\lambda_{stop} = \Delta v \, t_s$

  - $\lambda_{stop} < H$ for St < 3

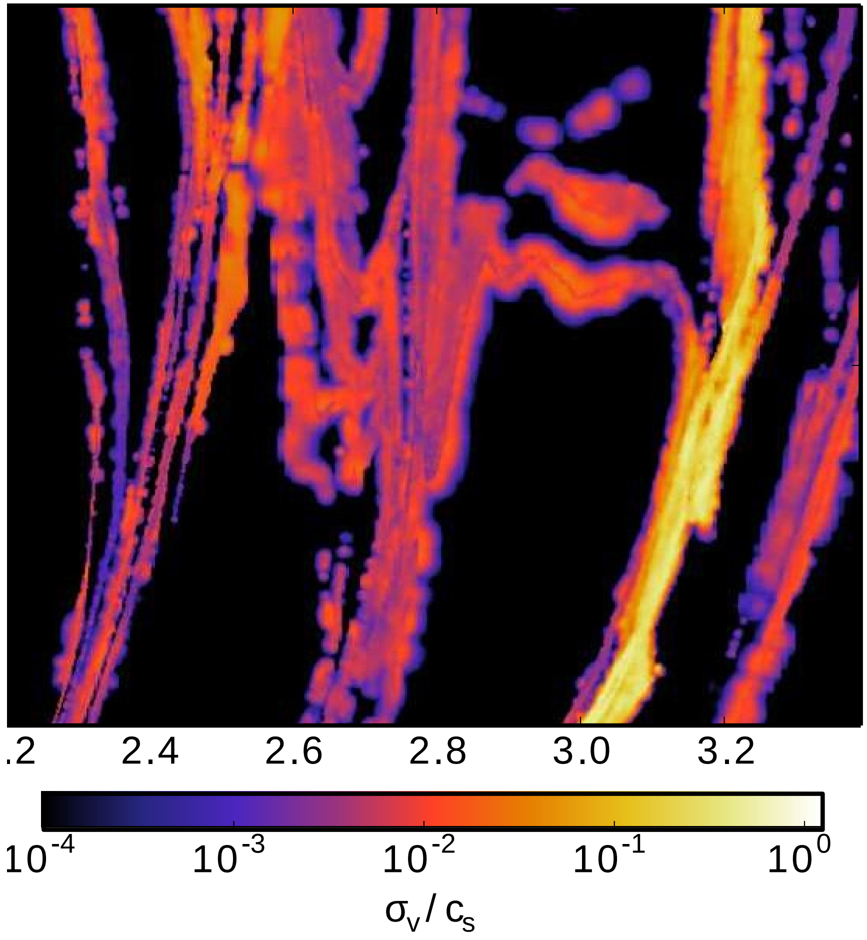
- Large St:
  - Gravitationally driven random walk

- Small St:
  - Gravity ineffective
  - What is driving?

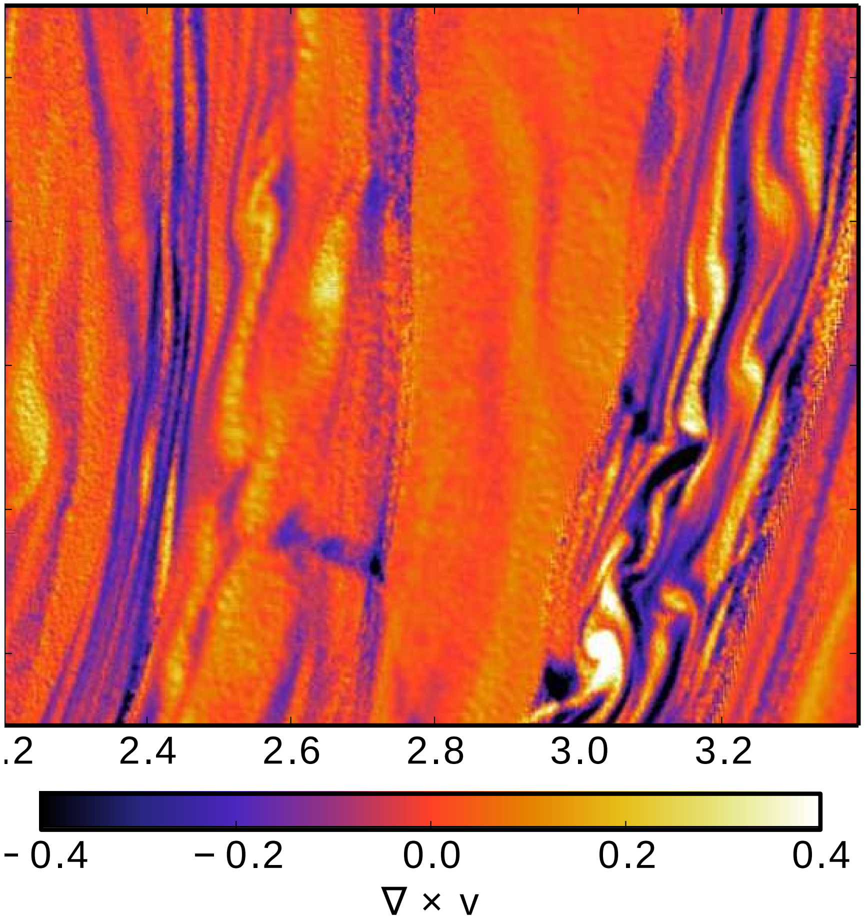


Relative velocities:

Inhomogeneity



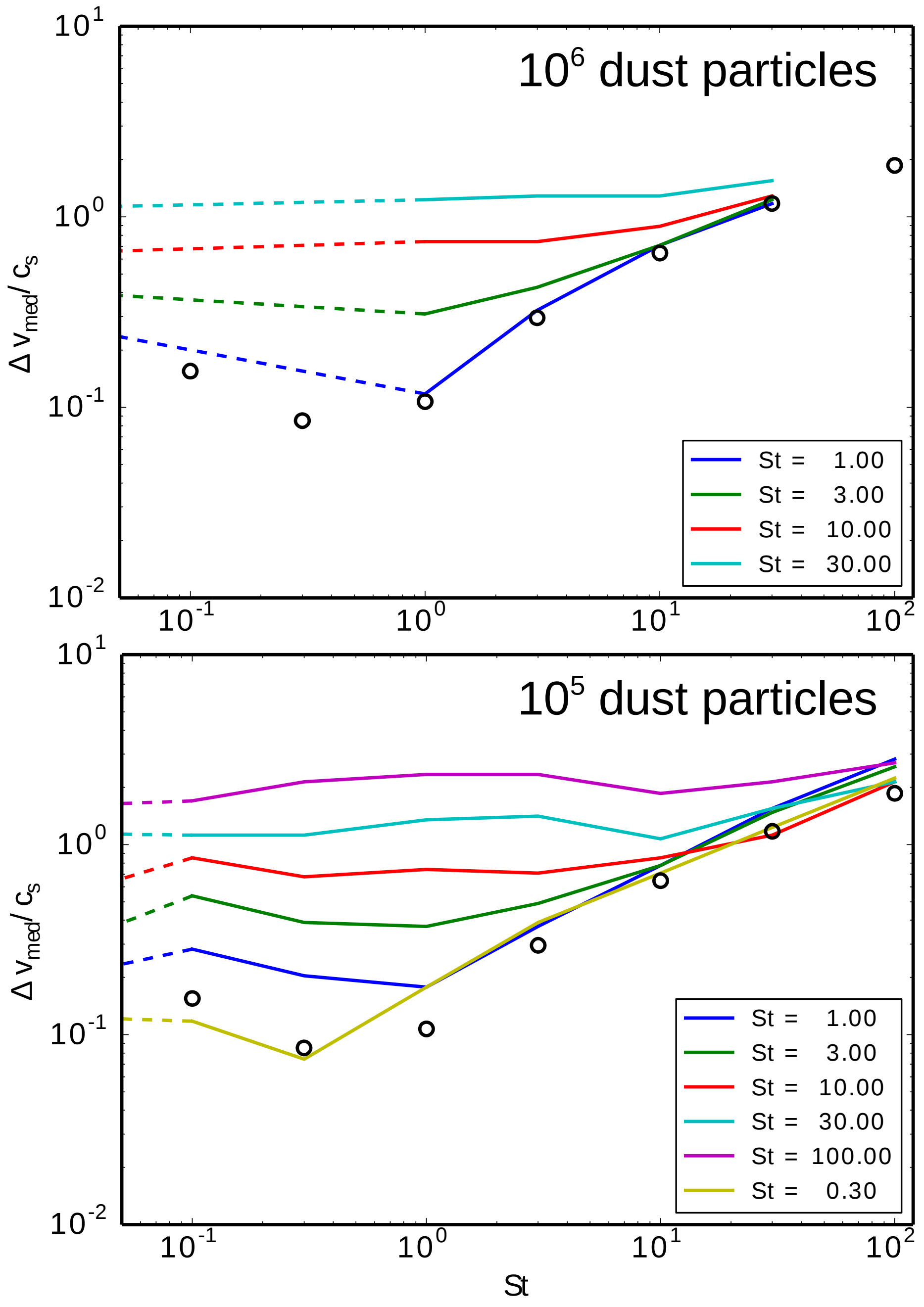
Dust Collision velocity



Gas vorticity

Bi-disperse case:

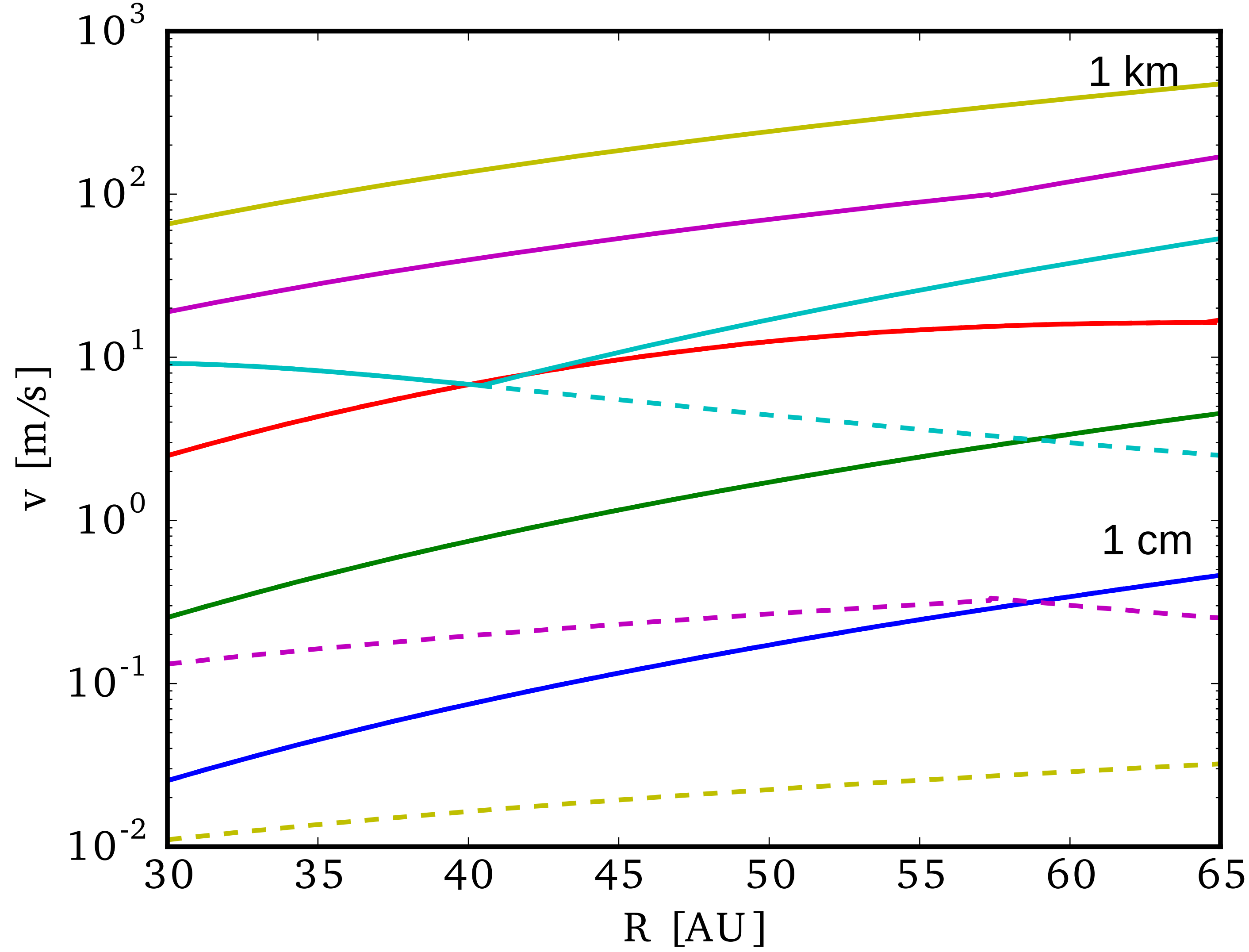
Velocity distribution



- Dominated by velocity dispersion if one particle large St
- Radial drift larger at low St

Physical units

Radial scaling: Collision velocity



Particle size:
1cm to 1km

Can constrain grain growth in self-gravitating discs

Planetesimal formation:
May be possible beyond 30 au

Summary

- Dust dynamics can be fundamentally different to gas
- Dust implementation in GANDALF (SPH) is under way
  - Include feed back on the gas
  - Hope to include dust in Meshless Finite Volume
  - Testing required!
- Standard tests are passed relatively easily
  - Proper testing in astrophysical context is important
- Accurate dust and gas simulations are possible
  - Many interesting prospects for ALMA observations